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Mat: 15/ENG07/026
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(1) $F(x) = e^{-0.5x}(4-x) - 2$

* Given Initial guess value of 0.5
* Maximum percentage absolute error = 1e-9

(i) Find the root of the function

Solution

$f(x) = (4-x)e^{-0.5x} - 2$

To find the root

When $x=0$, $f(x)=2$

$x=1$, $f(x) = -0.180$

$\therefore x \in (0, 1)$

$\therefore f(x) = (4-x)e^{-0.5x} - 2$

$f'(x) =$ Expanding

$4e^{-0.5x} - xe^{-0.5x} - 2$

$= (4x - 0.5)e^{-0.5x} - (x - -0.5)e^{-0.5x}$

$= -2e^{-0.5x} - (-0.5xe^{-0.5x} + e^{-0.5x})$

$= -2e^{-0.5x} + 0.5xe^{-0.5x} - e^{-0.5x}$

factorizing

$= (-2-1)e^{-0.5x} + 0.5xe^{-0.5x}$

$= -3e^{-0.5x} + 0.5xe^{-0.5x}$

$= e^{-0.5x}[0.5x - 3]$

$\therefore$ Using Newton-Raphson Method;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

* percentage absolute error $= \left[\frac{\text{final value} - \text{initial value}}{\text{final value}}\right] \times 100$

Percentage absolute error $= \left[\frac{x_{n+1} - x_n}{x_{n+1}}\right] \times 100$

for Iter 1;

let $x_n = 0.5$

$f(x_n) = (4 - 0.5)e^{-0.5 \times 0.5} - 2$

$= 0.7258027407$

$f'(x_n) = e^{-0.5 \times 0.5}[(0.5 \times 0.5) - 3]$

$= -2.1417$

$\therefore x_{n+1} = 0.5 - \dfrac{0.7258027407}{-2.141702153}$

$= 0.5 + \dfrac{0.7258027407}{2.141702153}$

$= 0.8388906061$

% absolute error $= (x_{n+1} - x_n) \times 100$

$= (0.8388906061 - 0.5) \times 100$

$= 33.88906061$

for Iter 2;

$x_n = x_{n+1}$

$= 0.8388906061$

$\therefore f(x_n) = (4 - 0.8388906061)e^{-0.5 \times 0.8388906061} - 2$

$= 0.07814929777$

$f'(x_n) = e^{-0.5 \times 0.8388906061}[(0.5 \times 0.8388906061) - 3]$

$= -1.696486032$

$\therefore x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$= 0.8388906061 - \dfrac{0.07814929777}{-1.696486032}$

$= 0.8388906061 + \dfrac{0.07814929777}{1.696486032}$

$= 0.8849714552$

% absolute error $= (0.8849714552 - 0.8388906061) \times 100$

$= 4.608084991$

for Iter 3 ;

$x_n = x_{n+1}$

$= 0.884971455 2$

$\therefore f(x_n) = (4 - 0.884971455 2)\, e^{-0.5 \times 0.884971455 2} - 2$

$= 0.001211085128$

$f'(x_n) = e^{-0.5 \times 0.884971455 2} [(0.5 \times 0.884971455 2) - 3]$

$= -1.643043201$

$\therefore x_{n+1} = 0.884971455 2 - \dfrac{0.001211085128}{-1.643043201}$

$= 0.884971455 2 + \dfrac{0.001211085128}{1.643043201}$

$= 0.885708554$

% absolute error $= (0.885708554 - 0.884971455 2) \times 100$

$= 0.07370988$

for Iter 4 ;

$x_n = x_{n+1}$

$= 0.885708554$

$\therefore f(x_n) = (4 - 0.885708554)\, e^{-0.5 \times 0.885708554} - 2$

$= 4.072736 \times 10^{-7}$

$f'(x_n) = e^{-0.5 \times 0.885708554} [(0.5 \times 0.885708554) - 3]$

$= -1.642200987$

$x_{n+1} = 0.885708554 - \dfrac{4.072736 \times 10^{-7}}{-1.642200987}$

$= 0.885708554 + \dfrac{4.072736 \times 10^{-7}}{1.642200987}$

$= 0.885708802$

% absolute error = $(0.885708802 - 0.884708551) \times 100$

$= 2.48 \times 10^{-5}$

for Iter 5

$x_n = x_{n+1}$

$= 0.885708802$

$f(x_n) = (4 - 0.885708802)e^{-0.3 \times 0.885708802} - 2$

$= 7.84 \times 10^{-12}$

$f'(x_n) = e^{-0.3 \times 0.885708802}[(0.3 \times 1.885708802)]$

$= -1.642200704$

$x_{n+1} = 0.885708802 - \dfrac{7.84 \times 10^{-12}}{-1.642200704}$

$= 0.885708802 + \dfrac{7.84 \times 10^{-12}}{1.642200704}$

$= 0.885708802$

% absolute error $= (0.885708802 - 0.885708802) \times 100$

$= 0$

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	% absolute error
1	0.5	0.7258027407	-2.1419	0.8388906061	33.88906061
2	0.8388906061	0.07514929777	-1.676486232	0.8849714552	4.608084921
3	0.8849714552	0.00131105128	-1.643043101	0.8857083554	0.073704886
4	0.8857083554	4.07936×10^{-7}	-1.642200787	0.885708802	2.48×10^{-5}
5	0.885708802	7.84×10^{-12}	-1.642200704	0.885708802	0

Matlab Code

Commandwindow

```
clear
clc
mtar = 0;
u = 18;
for i = (1:mt)
iter(i+1) = i;
ft = ((0.3*u(i)^2)/(500+(log(u(i)^3)))) - 0.02*u(i);
ft2 = (3*u(i)))/(5*(log(u(i))^3 + 500)) - (9*u(i)*log(u(i))^2)/(10*(log(u(i))^3 + 500)^2 - 1/50;
u(i+1) = u(i) - (ft/ft2);
ea = abs(u(i+1) - u(i))*100;
if ea(i+1) > 1e-15
break
end
end
tab = [iter 'u' 'ea'];
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