

If the model of a system of a system having thermocouples measuring temperature $T(^{\circ}\text{C})$ at its different points is given by the set of expressions in equation 1, estimate the values of the temperatures using.

- i. Gauss elimination method.
- ii. Gauss elimination method with the aid of excel
- iii. Matrix Inverse method with aid of excel.
- iv. Matrix Inverse method with the aid of MATLAB.

$$\left\{ \begin{array}{l} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_6 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_6 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{array} \right\} \quad \text{--- (1)}$$

Sol

$$\begin{array}{cccccc|c|c} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & T_2 & 20 \\ 1 & 3 & -3 & 1 & 2 & 1 & T_3 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & T_4 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & T_5 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & T_6 & -27 \end{array}$$

$$\begin{array}{cccccc|c|c} 1 & 1 & -2 & 1 & 3 & -1 & & 4 \\ (2 - (\frac{2}{1}) \times 1) & (-1 - (\frac{2}{1}) \times 1) & (1 - (\frac{2}{1}) \times -2) & (2 - (\frac{2}{1}) \times 1) & (1 - (\frac{2}{1}) \times 3) & (-3 - (\frac{2}{1}) \times -1) & & 20 - (\frac{2}{1}) \times 4 \\ 1 - (\frac{1}{1}) \times 1 & (3 - (\frac{1}{1}) \times 1) & (-3 - (\frac{1}{1}) \times -2) & (-1 - (\frac{1}{1}) \times 1) & (2 - (\frac{1}{1}) \times 3) & (1 - (\frac{1}{1}) \times -1) & & -15 - (\frac{1}{1}) \times 4 \\ 5 - (\frac{5}{1}) \times 1 & 2 - (\frac{5}{1}) \times 1 & (-1 - (\frac{5}{1}) \times -2) & (-1 - (\frac{5}{1}) \times 1) & (2 - (\frac{5}{1}) \times 3) & (1 - (\frac{5}{1}) \times -1) & & -3 - (\frac{5}{1}) \times 4 \\ (-3 - (\frac{-3}{1}) \times 1) & (-1 - (\frac{-3}{1}) \times 1) & (2 - (\frac{-3}{1}) \times -2) & (3 - (\frac{-3}{1}) \times 1) & (1 - (\frac{-3}{1}) \times 3) & (3 - (\frac{-3}{1}) \times -1) & & 16 - (\frac{-3}{1}) \times 4 \\ 4 - (\frac{4}{1}) \times 1 & (3 - (\frac{4}{1}) \times 1) & (1 - (\frac{4}{1}) \times -2) & (-6 - (\frac{4}{1}) \times 1) & (-3 - (\frac{4}{1}) \times 3) & (-2 - (\frac{4}{1}) \times -1) & & -27 - (\frac{4}{1}) \times 4 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & 1 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & 1 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 1 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & 1 & -43 \end{bmatrix}$$

Done with Excel

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & 1 & -35 \\ 0 & 0 & -0.67 & 6 & 6.667 & -0.67 & 1 & 36 \\ 0 & 0 & 7.33 & 10 & -13.33 & 2.33 & 1 & -47 \end{bmatrix}$$

Done with Excel

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & 1 & -16.14 \\ 0 & 0 & 0 & 5.42 & 5.42 & -0.28 & 1 & 32.85 \\ 0 & 0 & 0 & -3.71 & 0.28 & -1.85 & 1 & -12.43 \end{bmatrix}$$

Done with Excel

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & 0 & -2.57 & -6.57 & 4.71 & 1 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & 1 & -1.22 \\ 0 & 0 & 0 & 0 & 1.11 & -8.67 & 1 & 10.89 \end{bmatrix}$$

Done with Excel

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & -1.22 \\ 0 & 0 & 0 & 0 & 0 & -11.21 & 11.21 \end{bmatrix}$$

Final

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 \\ 0 & 0 & 0 & 0 & 0 & -11.21 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.14 \\ -1.22 \\ 11.21 \end{bmatrix}$$

$$-11.21 \times T_6 = 11.21$$

$$T_6 = -1$$

$$4.22 T_5 + 9.67 T_6 = -1.22$$

$$4.22 T_5 + 9.67(-1) = -1.22$$

$$T_5 = \frac{-1.22 + 9.67}{4.22}$$

$$T_5 = 2$$

$$\text{iii)} \quad 2.57 T_4 = -16.14 + 0.57(2) + 4.71(-1)$$

$$T_4 = \frac{-16.14 + 0.57(2) + 4.71(-1)}{-2.57}$$

$$T_4 = 4$$

$$T_4 = 4$$

$$iv) \quad 2.33T_3 - 2T_4 - 4.33T_5 + 1.33T_6 = -11$$

$$2.33T_3 = -11 + 2T_4 + 4.33T_5 - 1.33T_6$$

$$T_3 = \frac{-11 + 2(4) + 4.33(2) - 1.33(-1)}{2.33}$$

$$T_3 = 3$$

$$T_3 = 3$$

$$v) \quad -3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$$

$$T_2 = \frac{12 - 5T_3 - 5T_5 - T_6}{-3}$$

$$T_2 = \frac{12 - 5(3) - 5(2) - (-1)}{-3}$$

$$T_2 = -2$$

$$= T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 = 4 - T_2 - 2T_3 - T_4 - 3T_5 + T_6$$

$$T_1 = 4 - (-2) + 2(3) - 4 - 3(2) + (-1)$$

$$= 4 + 2 + 6 - 4 - 6 - 1$$

$$= 1$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = 1$$