

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 8T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

Transforming the equation to matrix

$$\begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \text{Row 4} \\ \text{Row 5} \\ \text{Row 6} \end{array} \begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{pmatrix}$$

Using a factor of (2) to multiply Row 1 and Subtract Row 2

$$2 - (2)(1); -1 - (2)(1); 1 - (2)(-2); 2 - (2)(1); 1 - (2)(3); -3 - (2)(-1); 20 - (2)(4)$$

$$0 \quad -3 \quad 5 \quad 0 \quad -5 \quad 1 \quad 12$$

Using a factor (1) to multiply Row 1 and Subtract Row 3

$$1 - (1)(1); 3 - (1)(1); -3 - (1)(-2); -1 - (1)(1); 2 - (1)(3); 1 - (1)(-1); -15 - (1)(4)$$

$$0 \quad 2 \quad -1 \quad 2 \quad -1 \quad 2 \quad -19$$

Using a factor (5) to multiply Row 1 and Subtract Row 4

$$5 - (5)(1); 2 - (5)(1); -1 - (5)(-2); -1 - (5)(1); 2 - (5)(3); 1 - (5)(-1); -3 - (5)(4)$$

$$0 \quad -3 \quad 9 \quad -6 \quad 13 \quad 6 \quad -23$$

Using a factor (-3) to multiply Row 1 and Subtract Row 5

$$-3 - (-3)(1); -1 - (-3)(1); 2 - (-3)(-2); 3 - (-3)(1); 1 - (-3)(3); 3 - (-3)(-1); 16 - (-3)(4)$$

$$0 \quad 2 \quad -4 \quad 6 \quad 10 \quad 0 \quad 28$$

Using a factor (4) to multiply Row 1 and Subtract Row 6

$$4 - (4)(1); 3 - (4)(1); 1 - (4)(-2); -6 - (4)(1); -3 - (4)(3); -2 - (4)(-1); -27 - (4)(4)$$

$$0 \quad -1 \quad 9 \quad -10 \quad -15 \quad 2 \quad -43$$



The matrix becomes:

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & -10 & -15 & 2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{pmatrix}$$

Using a factor of  $(-\frac{2}{3})$  to multiply Row 2 and subtract from Row 3  
 $2 - (-\frac{2}{3})(3); -1 - (-\frac{2}{3})(5); -2 - (-\frac{2}{3})(0); -1 - (-\frac{2}{3})(-5); 2 - (-\frac{2}{3})(-1); -19 - (-\frac{2}{3})(12)$   
 $0 \quad 2.3333 \quad -2 \quad -4.3333 \quad 1.3333 \quad -11$

Using a factor of (1) to multiply Row 2 and subtract Row 4  
 $-3 - (1)(-3); 9 - (1)(5); -6 - (1)(0); -13 - (1)(-5); 6 - (1)(-1); 23 - (1)(6)$   
 $0 \quad 4 \quad -6 \quad -8 \quad 7 \quad -35$

Using a factor of  $(-\frac{2}{3})$  to multiply Row 2 and subtract Row 5  
 $2 - (-\frac{2}{3})(-3); -4 - (-\frac{2}{3})(5); 6 - (-\frac{2}{3})(0); 10 - (-\frac{2}{3})(-5); 0 - (-\frac{2}{3})(-1); 28 - (-\frac{2}{3})(6)$   
 $0 \quad -0.6666 \quad 6 \quad 6.6666 \quad -0.6666 \quad 36$

Using a factor of  $(\frac{1}{3})$  to multiply Row 2 and subtract Row 6  
 $-1 - (\frac{1}{3})(-3); 9 - (\frac{1}{3})(5); -10 - (\frac{1}{3})(0); -15 - (\frac{1}{3})(-5); 2 - (\frac{1}{3})(-1); -43 - (\frac{1}{3})(12)$

Using 0  $7.3333 \quad -10 \quad -15.3333 \quad 2.3333 \quad -47$

The new matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -0.6666 & 6 & 6.6666 & -0.6666 \\ 0 & 0 & 7.3333 & -10 & -15.3333 & 2.3333 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{pmatrix}$$

Using a factor of 0.2856 to multiply Row 3 and subtract Row 5

$-0.6666 - (0.2856)(2.3333); 6 - (-0.2856)(-2); 6.6666 - (-0.2856)(-4.3333)$   
 $0 \quad 5.4285 \quad 5.4285$   
 $-0.6666 - (-0.2856)(1.3333); 36 - (-0.2856)(-11)$   
 $-0.28571 \quad 32.85714$

Using a factor of  $(7.3333/2.3333)$  to multiply Row 2 and subtract Row 6



$$\begin{aligned}
 &7.8583 - (3.1428)(2.3333); -10 - (3.1428)(-2); -13.3333(3.1428)(-4.3333) \\
 &\quad 0 \quad \quad \quad -3.71429 \quad \quad \quad 0.2857 \\
 &2.3333 - (3.1428)(1.3333); -47 - (3.1428)(-11) \\
 &\quad -1.85714 \quad \quad \quad -12.4286
 \end{aligned}$$

The new matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.571 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 5.4285 & 5.4285 & -0.2857 \\ 0 & 0 & 0 & -3.7142 & 0.2857 & -1.8571 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ 32.8571 \\ -12.4286 \end{pmatrix}$$

Using a factor  $(5.4285 / -2.571)$  to multiply row 4 and subtract from row 5

$$\begin{aligned}
 &5.4285 - (-2.1114)(-2.571); \quad 5.4285 - (-2.1114)(-0.5714); \quad -0.2857 - (-2.1114)(4.7142) \\
 &\quad 0 \quad \quad \quad 4.2222
 \end{aligned}$$

$$\begin{aligned}
 &-0.2857 - (-2.1114)(4.7142); \quad 32.8571 - (-2.1114)(-16.1429) \\
 &\quad 9.6666 \quad \quad \quad -1.2222
 \end{aligned}$$

Using a factor of  $(-3.7142 / -2.571)$  to multiply row 5 and divide row 6

$$\begin{aligned}
 &-3.9142 - (1.4446)(-2.571); \quad 0.2857 - (1.4446)(-0.5714); \\
 &\quad 0 \quad \quad \quad 1.1111
 \end{aligned}$$

$$\begin{aligned}
 &-1.8571 - (1.4446)(4.7142); \quad -12.4286 - (1.4446)(-16.1429) \\
 &\quad -8.6666 \quad \quad \quad 10.8888
 \end{aligned}$$

The new matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 0 & 4.2222 & 9.6666 \\ 0 & 0 & 0 & 0 & 1.1111 & -8.6666 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 10.8888 \end{pmatrix}$$

Using a factor of  $(1.1111 / 4.2222)$  to multiply row 5 and subtract row 6

$$\begin{aligned}
 &1.1111 - (0.2631)(4.2222); \quad -8.6666 - (0.2631)(9.6666); \quad 10.8888 - (0.2631)(-1.2222) \\
 &\quad 0 \quad \quad \quad -11.2105 \quad \quad \quad 11.2105
 \end{aligned}$$



The New Matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 5 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2.5714 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -4.3333 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 4.2222 & 9.6666 & 2 \\ 0 & 0 & 0 & 0 & 0 & -11.2105 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 11.2105 \end{pmatrix}$$

else we back substitution.

$$-11.2105 T_6 = 11.2105$$

$$T_6 = \frac{11.2105}{-11.2105} = -1$$

$$4.2222 T_5 + 9.6666 T_6 = -1.2222$$

$$T_5 = \frac{-1.2222 - 9.6666(-1)}{4.2222} = 2$$

$$-2.5714 T_4 - 0.5714 T_5 + 4.7142 T_6 = -16.1429$$

$$T_4 = \frac{-16.1429 - 4.7142(-1) + 0.5714(2)}{-2.5714} = 4$$

$$2.3333 T_3 - 2 T_4 - 4.3333 T_5 + 1.3333 T_6 = -11$$

$$T_3 = \frac{-11 - 1.3333(-1) + 4.3333(2) + 2(4)}{-2.3333} = 3$$

$$\Rightarrow -3 T_2 + 5 T_3 + 6 T_4 - 5 T_5 - T_6 = 12$$

$$T_2 = \frac{12 + (-1) + 5(2) - 5(3) - (-2)}{-3} = -2$$

$$\Rightarrow T_1 + T_2 - 2 T_3 + T_4 + 3 T_5 - T_6 = 4$$

$$T_1 = \frac{4 + (-1) - 3(2) - 4 + 2(3) - (-2)}{1} = 1$$

Therefore

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$