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15/ENG02/008
ENG 382

i) If the Maximum Percentage absolute error is desired to be 1E-9 using the Newton Raphson iteration method and initial guess value of 0.5, find the root of the function given in the equation below

$$f(x) = e^{-0.5x}(4-x)-2$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = e^{-0.5x_i}(4-x_i)-2$$

$$f'(x_i) =$$

$$u = e^{-0.5x} \qquad v = 4-x$$

$$du = 0.5e^{0.5x} \qquad dv = 1$$

$$f'(x_i) = (4-x)(-0.5e^{-0.5x}) - e^{-0.5x}$$

$$\therefore x_{i+1} = x_i - \frac{e^{-0.5x_i}(4-x_i)-2}{(4-x_i)(-0.5e^{-0.5x_i})-e^{-0.5x_i}}$$

at iter = 0 $\qquad x = 0.5$

for iter = 1

$$x_i = 0.5$$

$$x_{i+1} = 0.5 - \frac{e^{-0.5(0.5)}(4-0.5)-2}{(4-x_i)(-0.5e^{-0.5}\times 0.5)-e^{-0.5\times 0.5}}$$

$$x_{i+1} = 0.838890606$$

$$\text{error} \left| \frac{0.833890606 - 0.5}{0.8338906060} \right| \times 100$$

$$= 40.39747328$$

for iter 2

$x_1 = 0.888890606$

$$x_{i+1} = 0.838890606 - \frac{e^{-0.3(0.838890600)}(4-0.838890606) - 2}{(4-0.838890606)\times(0.5e^{(-0.3\times0.838890606)}) - e^{-0.5\times0.838890}}$$

$x_{i+1} = 0.884956000$

$$error = \left|\frac{0.884956000 - 0.838890606}{0.884956000}\right| \times 100$$

$= 5.205388064\,\%$

for iter 3

$x_1 = 0.884956000$

$$x_{1+1} = 0.884956000 - \frac{e^{-0.5(0.884956000)}(4-0.884956000) - 2}{(4-0.884956000)\times(0.5e^{-0.3\times0.884956000}) - e^{-0.3\times0.884956000}}$$

$x_{i+1} = 0.885708605$

$$error = \left|\frac{0.885708605 - 0.884956000}{0.885708604}\right| \times 100$$

$error = 0.084971964\,\%$

for iter 4

$x_1 = 0.885708605$

$$x_{i+1} = 0.885708604 - \frac{e^{-0.5(0.885708604)}(4-0.885708604) - 2}{(4-0.88570804)\times(-0.5e^{-0.5(0.885708604)}) - e^{0.5(0.885768604)}}$$

$x_{i+1} = 0.885708802$

$$error = \left|\frac{0.885708802 - 0.885708605}{0.885708804}\right| \times 100$$

$= 0.0000002224\%$

$= 2.224207319 \times 10^{-7}\%$

for iter 5

$x_i = 0.885708802$

$$x_{i+1} = 0.885708802 - \frac{e^{-0.3(0.885708802)}(4-0.885708802)-2}{(4-0.885708802)\times(0.5e^{-0.885708802\times0.3}) - e^{-0.885708802\times0.3}}$$

$= 0.885708802$

$$\text{error} = \left|\frac{0.885708802 - 0.885708802}{0.885708802}\right| \times 100$$

$= 0$

iter	x	error (%)
0	0.5	
1	0.838890606	40.39747328
2	0.884956000	5.205388064
3	0.885708605	0.084991964
4	0.885708802	$2.224207319 \times 10^{-7}$
5	0.885708802	0

The root of the equation is 0.885708802

2) A flat plate of mass m falling in air with velocity v is subjected to a downward gravitational force and an upward frictional drag force due to air. If the drag force, F is given by the equation below

$$F_D = \frac{0.3V^2}{500+(\ln V)^3} - 0.02X$$

and the terminal velocity is reached when the drag force is equal to gravitational force, that is

$$F_D = mg$$

taking $m = 3.5kg$ and $g = 9.8m/s^2$ with an initial guess value of $v = 18m/s$

Using newton raphson's method, Find the value of V.

Solution

$$F_D = mg$$
$$= 3.5 \times 9.8$$
$$= 34.3$$

$$34.3 = \frac{0.3v^2}{500+(\ln v)^3} - 0.02X$$

$$f(v) = \frac{0.3v^2}{500+(\ln v)^3} - 0.02X - 34.3$$

$$V_{i+1} = V_i - \frac{f(v_i)}{f'(v_i)}$$

$$f(v_1) = \frac{0.3v_1^2}{500+(\ln v_1)^3} - 0.02v_1 - 34.3$$

$$f'(v_1) = \frac{d}{dv}\left[\frac{0.3v_1^2}{500+(\ln(v_1))^3}\right] - 0.02$$

$$\frac{d}{dv}\left[\frac{0.3V_i^2}{500+(\ln(V_i))^3}\right] =$$

$a = 0.3V_i^2$ and $b = 500+(\ln(V_i))^3$

$\frac{da}{dv} = 0.6V_i^2$ $\quad \frac{db}{dv} = 3(\ln(V_i))^2\left(\frac{1}{V}\right)$

$$= \frac{b\,{}^{da}/_{dv} - a\,{}^{db}/_{dv}}{b^2}$$

$$= \frac{[500+(\ln(V_1))^3(0.6V_1^2)] - [0.3V_1^2(3\ln(V_1)^2(1/v)]}{[500+(\ln(V_1)^2]^2} - 0.02$$

$$\therefore V_{i+1} = V_1 - \frac{\left[\frac{0.3V_1^2}{500+(\ln V_1)^3} - 0.02V_1 - 34.3\right]}{\left(\frac{[500+(\ln(V_1)^3(0.6V_1)] - [0.3V_1^2((3\ln(V_1)^2(1/v))}{[500+(\ln(V_1)^3]^2}\right)} - 0.02$$