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15/01/2024/0.86
Elect/240

$$\begin{aligned} 1) \quad T_1 + T_2 - T_3 + T_4 + 3T_5 - T_6 &= 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 &= 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 &= -15 \\ 5T_1 + 2T_2 - T_3 - T_4 - 12T_5 + T_6 &= -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 - 5T_6 &= 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 &= -27 \end{aligned}$$

Soln

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 5 \\ 4 & 3 & 1 & -6 & 3 & -2 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{matrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{matrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 5 & 16 \\ 4 & 3 & 1 & -6 & 3 & -2 & -27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -1 & 1 & 2 & 1 & -3 & 20 \\ 0 & 0 & -3 & -1 & 2 & 1 & -15 \\ 0 & 0 & 0 & -1 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 0 & 0 & -2 & -27 \end{bmatrix}$$

$$i) 2 - 2(1) = 0$$

$$ii) 1 - 2(3) = -5$$

$$iii) -3 - 2(-1) = -1$$

$$iv) 20 - 2(4) = 12$$

$$\text{Row 2} \rightarrow (0 \ -3 \ 5 \ -1 \ 12)$$

$$a_{31} \rightarrow 0 \text{ Row 2} = \frac{1}{-3} (\text{Row 2})$$

$$i) 1 - 1(1) = 0$$

$$ii) 3 - 1(-1) = 2$$

$$iii) -3 - 2(-2) = 1$$

$$iv) 2 - 1(1) = 1$$

$$v) 2 - 1(3) = -1$$

$$vi) 1 - (-1) = 2$$

$$vii) -15 - 1(4) = -19$$

$$\text{Row 3} \rightarrow [0 \ 2 \ -1 \ -2 \ -1 \ 2 \ -19]$$

$$a_{41} \rightarrow 0 \text{ Row 4} = \frac{5}{2} (\text{Row 3})$$

$$i) 5 - 5(1) = 0$$

$$ii) 2 - 5(3) = -13$$

$$iii) -1 - 5(-2) = 9$$

$$iv) -1 - 5(1) = -6$$

$$v) 2 - 5(3) = -13$$

$$vi) 1 - 5(-1) = 6$$

$$vii) -3 - 5(4) = -23$$

$$(\text{Row 4}) \rightarrow (0 \ -3 \ 9 \ -6 \ 15 \ 6 \ -23)$$

$$a_{51} \rightarrow (\text{Row 5}) = \frac{1}{-3} (\text{Row 4})$$

$$(\text{Row 3}) + 3 (\text{Row 5})$$

$$i) -3 + 3(1) = 0$$

$$ii) -1 + 3(3) = 8$$

$$iii) 2 + 3(-2) = -4$$

$$iv) 3 + 3(1) = 6$$

$$v) 1 + 3(3) = 10$$

$$vi) 3 + 3(-1) = 0$$

(Row 5)

$$i) 4 -$$

$$ii) 3 -$$

$$iii) 1 -$$

$$iv) -6 -$$

$$v) -$$

$$vi) -$$

$$vii) -$$

$$(Row 5) \rightarrow \begin{bmatrix} 0 & 2 & -4 & 6 & 10 & 0 & 28 \end{bmatrix}$$

$$a_{61} \rightarrow 0 \quad (row 6) - \frac{4}{1} (row 1)$$

$$\begin{aligned} \text{① } 4 - 4(1) &= 0 \\ \text{② } 3 - 4(1) &= -1 \\ \text{③ } 1 - 4(-2) &= 9 \\ \text{④ } -6 - 4(1) &= -10 \\ \text{⑤ } -3 - 4(3) &= -15 \\ \text{⑥ } -3 - 4(-1) &= 2 \\ \text{⑦ } -27 - 4(4) &= -43 \end{aligned}$$

$$Row 6 \rightarrow (0, -1, 9, -10, -15, 3, 43)$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -14 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{bmatrix}$$

To make $a_{32} = 0$ we have $row 3 - \frac{a_{32}}{a_{22}} \times (row 2)$

$$\begin{aligned} \text{① } 2 - \frac{2}{-3}(-3) &= 0 \\ \text{② } -1 - \frac{2}{-3}(5) &= \frac{1}{3} \\ \text{③ } -2 - \frac{2}{-3}(0) &= -2 \\ \text{④ } -1 - \frac{2}{-3}(-5) &= \frac{7}{3} \\ \text{⑤ } 2 - \frac{2}{-3}(-1) &= \frac{4}{3} \\ \text{⑥ } -17 - \frac{2}{-3}(12) &= -11 \end{aligned}$$

$$The \text{ new } row 3 = \left(0, 0, \frac{1}{3}, -2, -\frac{13}{3}, \frac{4}{3}, -11 \right)$$

To make $a_{42} = 0$ we had $row 4 - \left(\frac{3}{-3} \right) row 2 = row 4 + row 2$

$$\begin{aligned} \text{① } -3 + (-3) &= 0 \\ \text{② } 4 + 1(5) &= 9 \\ \text{③ } -6 + 1(0) &= -6 \\ \text{④ } -3 + 1(-5) &= -8 \\ \text{⑤ } 6 + 1(-1) &= 5 \\ \text{⑥ } -23 + 1(12) &= -11 \end{aligned}$$

New Row 4 = $(0, 0, 4, -6, -8, 7, 35)$

To make $a_{55} = 0$, we have $\text{Row } 5 = -2/3 \times \text{Row } 4 + \text{Row } 1$

$$\text{I) } 2 + 2/3(-3) = 0$$

$$\text{II) } -4 + 2/3(5) = -2/3$$

$$\text{III) } 6 + 2/3(0) = 6$$

$$\text{IV) } 10 + 2/3(-5) = 20/3$$

$$\text{V) } 6 + 2/3(-1) = 2/3$$

$$\text{VI) } 28 + 2/3(12) = 36$$

The new row 5 = $(0, 0, -2/3, 6, 24/3, -2/3, 36)$

To make $a_{66} = 0$, we have $\text{Row } 6 = 1/3 \times \text{Row } 5$

$$\text{I) } -1 - 1/3(-2) = 0$$

$$\text{II) } 4 - 1/3(5) = 22/3$$

$$\text{III) } -10 + 1/3(0) = -10$$

$$\text{IV) } -15 - 1/3(-5) = -40/3$$

$$\text{V) } 2 - 1/3(-1) = 7/3$$

$$\text{VI) } -43 - 1/3(12) = -47$$

The new row 6 = $(0, 0, 22/3, -10, 40/3, 7/3, -47)$

Thus the new set of augmented matrix is

$$\left[\begin{array}{cccccc|c} 1 & -2 & 1 & 3 & -1 & 4 & 1 \\ 0 & -3 & 5 & 6 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -17/3 & 4/3 & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & -35 \\ 0 & 0 & -2/3 & 6 & 24/3 & -2/3 & 36 \\ 0 & 0 & 22/3 & -10 & 40/3 & 7/3 & -47 \end{array} \right]$$

To make $a_{43} = 0$, we have

$\text{Row } 4 = (4/7) \times \text{Row } 3 + \text{Row } 4 \times 4/7$

$$\text{I) } 4 - \frac{4 \times 7}{7} \left(\frac{7}{3} \right) = 0$$

$$\text{II) } -6 - \left(\frac{12}{7} \right) \left(\frac{3}{3} \right) = -18/7$$

$$\text{III) } -8 - \left(\frac{12}{7} \right) \left(\frac{3}{3} \right) = -4/7$$

$$\text{IV) } 7 - \left(\frac{12}{7} \right) \left(\frac{4}{3} \right) = 37/7$$