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16/ENG01/005
Chemical Engineering
ENG 282 – Engineering mathematics II

Solution

a) Let y be the amount of bacteria present at time t.
The rate of change of y with time is proportional to the amount of bacteria present at time t. i.e y

$$\frac{dy}{dt} \propto y \Rightarrow \frac{dy}{dt} = ky$$

where k is a constant.

$$\frac{dy}{y} = k\,dt$$

$$\int \frac{1}{y}\,dt = k\int dt$$

$$\ln y = kt + c \Rightarrow e^{\ln y} = e^{kt+c}$$

$$y = e^{kt} \cdot e^{c} \quad \text{let } e^{c} = b$$

$$y = be^{kt}$$

Initial amount of bacteria ie $y(0) = 20$

$$\Rightarrow 20 = be^{k \cdot 0}$$

$$\therefore b = 20$$

$$y = 20e^{kt}$$

b) $1\frac{1}{2}$ days = 36 hours

halflife = 5 hours

when $t = H$, the bacteria is twice it's original number.

$$be^{kt} = be^{kH} = 2 \cdot y$$

$$20e^{5k} = 2 \cdot 20$$

$$20e^{5k} = 40 \Rightarrow e^{5k} = \frac{40}{20} = 2$$

$$k = \frac{\ln 2}{5}$$

$$k = 0.1386$$

$y = 20e^{0.1386 \times 36}$

$y = 2937.5$

$y \simeq 2938$

d) at $t = 0$, $y = b$

$\therefore$ ~~at, t = 0~~ $y(0) = 10$

~~$y = 10e^{kt}$~~ $10 = be^{k \cdot 0}$

$b = 10$

$\Rightarrow y = 10e^{kt}$

$10e^{k5} = 20$

$e^{k5} = \frac{20}{10} = 2$

$k = \frac{\ln 2}{5}$

$k = 0.1386$

$y = 10e^{0.1386t}$

at $y = 10e^{0.1386 \times 36}$

$y = 1468.7$

$y \simeq 1469$

at $t = 0$, $y = b$

$(y(0)) = 30$

$30 = be^{k \cdot 0}$

$b = 30$

$\Rightarrow y = 30e^{kt}$

$30e^{k5} = 60$

$e^{k5} = \frac{60}{30} = 2$

$K = \frac{\ln 2}{5} = 0.1386$

$y = 30e^{0.1386t}$

at $t = 36$ hours

$y = 30e^{0.1386 \times 36}$

$y = 4406$

$y(0) = 50$

$50 = be^{k \cdot 0}$

$b = 50$

$\Rightarrow y = 50e^{kt}$

$50e^{kH} = 100$

$e^{kH} = \frac{100}{50} = 2$

$k = \frac{\ln 2}{5} = 0.1386$

$\Rightarrow y = 50e^{0.1386t}$

$y = 50e^{0.1386 \times 36}$

$y = 7343.8$

$y \simeq 7344$

2) from the graph obtained in step d, it is shown that the amount of bacteria varies linearly with time. ie an increase in time, leads to an exponential growth of bacteria.