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ENG 282 Assignment 1.

An experiment is carried out by a biomedical engineer using a certain type of bacteria that doubles in population every 5hr in a growth medium. If the experiment is commenced with 20 bacteria,

- develop a model for the system,
- Use the model to estimate the population of the bacteria in  $1\frac{1}{2}$  days.
- With the aid of Microsoft Excel, simulate the model and plot the variation of the number of bacteria with time ( $t$ ) for  $t=0$  to  $t=15$ hr using a step time of 0.25hr.
- making the initial number of bacteria to be 10, 30 and 50 successively, plot the variations of the number of bacteria with time for  $t=0$  to  $t=80$ hr using a step time of 0.5hr on the same graph, and
- comment on the results obtained in (d).

Solution

a) Model for the system.

$$\begin{aligned}\frac{dy}{dt} &= ky & \text{but } k &= \frac{1}{5} \text{ and } \frac{1}{k} = e \\ \Rightarrow \int \frac{1}{y} dy &= \int k dt \\ \ln y &= k \int dt \\ \ln y &= kt + c \\ \ln y &= e(kt + c) \\ y &= e^{kt+c} \\ y &= e^{kt} \cdot e^c \\ \text{let } e^c &= y_0 \\ y &= y_0 e^{kt} \quad \text{--- (1)}\end{aligned}$$

At the beginning of the experiment,

$$t=0 \text{ and } y=20$$

$$20 = y_0 e^{k(0)}.$$

$$20 = y_0 \quad \text{--- (2)}$$

At  $t=5$ hrs.

$$y = 2y_0$$

$$y = 2 \times 20 = 40 \quad \text{--- (3)}$$

Inputting the value of  $y_0$  and  $y$  in (1),

$$40 = 20e^{kt}$$

$$\frac{40}{20} = e^{5k}$$

$$2 = e^{5k}$$

$$\ln 2 = 5K$$

$$0.6931 = 5K$$

$$K = \frac{0.6931}{5} = 0.13863$$

$$\therefore \text{model} \Rightarrow y = 20e^{0.13863t}$$

b) Estimated population of bacteria in  $1\frac{1}{2}$  days

$$1\frac{1}{2} \text{ days} = 24 \text{ hrs} + 12 \text{ hrs}$$

$$t = 36 \text{ hrs}$$

$$y = 20e^{0.13863(36)}$$

$$= 147.036 \times 20 = 2940 \text{ bacteria}$$

d) if  $y_0 = 10$

$$y = 10e^{Kt}$$

$$y = 10e^{0.13863t}$$

when  $y_0 = 30$

$$y = 30e^{Kt}$$

$$= 30e^{0.13863t}$$

when  $y_0 = 50$

$$y = 50e^{Kt}$$

$$= 50e^{0.13863t}$$

time ;  $t = 0$  to  $t = 30 \text{ hr}$ .