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16/ENG 07/018
PETROLEUM ENGINEERING.
ENG 282
ASSIGNMENT II

Solution

Step 1: Setting up a model

Let $F_A(t)$ denote the amount of fresh air in the room at time t.

By Balance law:

$$\frac{dF_A}{dt} = \text{Fresh Air inflow rate} - \text{Fresh Air outflow rate}$$

Input of Fresh Air = 600 ft^3/min

Initially there was no Fresh air.

Hence, $F_A(0) = 0$

Also

Output of mixture = 600 ft^3/min.

Mixture of Fresh Air and Normal Air = 20,000 ft^3/min

$$\frac{dF_A}{dt} = 600 - \frac{600}{20,000} \times F_A(t)$$

$$\frac{dF_A}{dt} = 600 - 0.03F_A$$

$$\frac{dF_A}{dt} = -0.03(F_A - 20,000)$$

Step 2: Solution of the Model:

$$\frac{dF_A}{dt} = -0.03(F_A - 20,000)$$

$$\frac{dF_A}{F_A - 20{,}000} = -0.03\,dt$$

Integrate both sides

$$\int \frac{dF_A}{F_A - 20{,}000} = \int -0.03\,dt$$

$$\ln(F_A - 20{,}000) = -0.03t + C$$

Taking ln of both sides

$$F_A - 20{,}000 = Ce^{-0.03t}$$

where $C = e^{c}$

$$F_A = 20{,}000 + Ce^{-0.03t}$$ (General Solution)

Initially, there was no fresh air

Hence

$$F_{A(0)} = 0$$

Recall

$$F_A = 20{,}000 + Ce^{-0.03t}$$

where $t = 0$, $F_A = 0$

$$0 = 20{,}000 + C \times e^{-0.03(0)}$$

$$C = -20{,}000$$

Therefore, sub. for C

$$F_{A(t)} = 20{,}000 - 20{,}000\,e^{-0.03t}$$ (Particular Solution)

b) Time at which 90% of the air will become fresh

$$\frac{90}{100} \times \frac{20{,}000}{1} = 20{,}000 - 20{,}000\,e^{-0.03t}$$

$$18{,}000 = 20{,}000 - 20{,}000\,e^{-0.03t}$$

$$-2{,}000 = -20{,}000\,e^{-0.03t}$$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$-2.3026 = -0.03t$

$t = 76.75$ mins

0.75 mins to seconds

$= 0.75 \times 60$

$= 45$ Seconds

Therefore $t = 76$ minutes 45 Seconds

c) 6 hours to minutes $= 60 \times 60 = 360$ minutes.

d) The steady-state Value of the amount of fresh air in the room $= 20{,}000$ (ft^3 of air).

e) The steady-state Value of the amount of fresh air in the room obtained from the response (graph) is given as a straight line where there is no longer increase in the amount of fresh air even though there is still increase in the time.

Here, the amount of fresh air in the room is steady (it does not change) with increase in Time (minutes).