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16/MHS01/029
Computer Engineering
Assignment 2

Answer

Let $y(t)$ be the amount of fresh air present at time t

y' = Fresh air inflow rate − fresh air outflow rate

(a) The room contains $20000 ft^3$ of air

Outflow rate = $600 ft^3/min$

$$\frac{600}{20000} = 0.03$$

This means that $0.03 ft^3$ of the total air flows out per minute

inflow rate = $600 ft^3/min$

$$y' = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03(y - 20000) \quad (i.e\ y' = -0.03(y-20000))$$

$$\int \frac{dy}{y-2000} = \int -0.03\,dt$$

$$\ln(y - 20000) = 0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

(Let $e^C = a$)

$$y - 20000 = e^{-0.03t} \cdot e^C$$

$$y - 20000 = ae^{-0.03t}$$

$$y(t) = 20000 + ae^{-0.03t}$$

Initially, there is no fresh air in the room

$$y(t) = 20$$

$$0 = 20000 + ae^{-0.03 \times 0}$$

$$0 = 20000 + a$$

$$a = -20000$$

$$y(t) = 20000 - 20000e^{-0.03t}$$

(b) 90% of the air in the room

$\frac{90}{100} \times 20000$

$= 18000$

$\therefore y(t) = 18000$

$y(t) = 20000 - 20000e^{-0.03t}$

$18000 = 20000 - 20000e^{-0.03t}$

$20000e^{-0.03t} = 20000 - 18000$

$e^{-0.03t} = \frac{2000}{20000}$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln 0.1}{-0.03}$

$t = 76.75$

$t \simeq 77$ minutes

(c) Done in the graph

(d) From the graph, steady state value $\simeq 20000 ft^3$

(e) It is at this point that an increase in time will not lead to an increase in amount of fresh air in the room