ENG 282 MATHS ASSIGNMENT 15/03/18

16/ENG06/063
MECHANICAL ENGINEERING
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It was discovered that 600ft^3/min of fresh air flows into a room containing 20000ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet (cfm). If the room contains no fresh air initially.

a) develop a model for the amount of fresh air at any time t in the room.

b) Calculate the time at which 90% of the air in the room will have become fresh.

c) With the aid of Microsoft Excel, Plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6hr$ using a step time of 5min. The response (graph) should be made to occupy an entire sheet alone.

d) Using the dynamic response plotted in (c), determine the steady-state value of the amount of fresh air in the room, and

e) Comment on the result obtained in (d).

Solution

(a) Let y represent fresh air

But,

Rate of Accumulation = Rate of Inflow − Rate of Outflow

$$\frac{dy}{dt} = 600 - 0.03y$$

rate of Inflow = $600\ ft^3/min$

rate of Outflow = $\frac{600}{20000} \times y = 0.03y$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{dy}{(y - 20000)} = -0.03dt$$

$$\ln(y - 20,000) = -0.03t + C$$

$$y - 20,000 = e^{-0.03t + C}$$

$$y - 20,000 = y_0 e^{-0.03t}$$

$$y = 20000 + y_0 e^{-0.03t}$$

If the room contained no fresh air $y = 0$, $t = 0$

$$0 = 20000 + y_0$$

$$y_0 = -20000$$

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t})$$

This is the model for the amount air at any time t.

b) Room contains 20000 ft^3 of air

90% of 20000

$= 18000\ ft^3/min$

from the model

$y = 20000(1 - e^{-0.05t})$

$18000 = 20000(1 - e^{-0.05t})$

$e^{-0.03t} = \left[\frac{18000}{20000} - 1\right]$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$-0.03t = -2.303$

$t = \frac{-2.303}{-0.03}$

$t = 76.77\ min$.

d) from the dynamic response plotted, the steady-state value of the amount of fresh air in the room is 20,000 ft^3 of air.

e) It was noticed that the the value of amount of fresh air steadily increase until it got to 20,000 ft^3 of air. Therefore despite the increase in time the amount of fresh air remained 20,000 ft^3 giving the steady-state value. In conclusion, 20,000 ft^3 of air is the maximum air for the room