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ELECTRICAL / ELECTRONICS

ENG 282 [ASSIGNMENT]

Answer

SETTING UP THE MODEL

Let $F_{in}(t)$ denote the amount of fresh air in the room at time t

Using balance law:

$$\frac{dF_{in}}{dt} = \text{fresh air inflow rate} - \text{Fresh air and air flow rate}$$

Input of fresh air = 600 ft³/min

Initially there was no fresh air

Hence, $F_{in}(0) = 0$

Also

Output of mixture = 600 ft³/min
 mixture of fresh air and normal air = 20,000 ft³/min

$$\frac{dF_{in}}{dt} = 600 - \frac{600}{20,000} F_{in}(t)$$

$$\frac{dF_{in}}{dt} = 600 - 0.03 F_{in}$$

$$\frac{dF_{in}}{dt} = -0.03 (F_{in} - 20,000)$$

For the model

$$= -0.03 (F_{in} - 20,000)$$

$$\frac{dF_{in}}{dt} = -0.03 dt$$

$$F_{in} - 20,000$$

Integrating both sides, we have

$$\int \frac{dF_{in}}{F_{in} - 20,000} = \int -0.03 dt$$

$$\ln(F_A - 20,000) = -0.03t + C$$

Taking \ln of both sides

$$F_A - 20,000 = e^{-0.03t} \cdot e^C$$

Let $C = e^C$

$$F_A = 20,000 + C \cdot e^{-0.03t}$$

Initially, there was no fish in the lake

Hence $F_A(0) = 0$

$$F_A = 20,000 + C \cdot e^{-0.03t} \quad \dots \quad (1)$$

when $t=0$, $F_A=0$

$$0 = 20,000 + C \cdot e^{-0.03(0)}$$

$$0 = 20,000 + C$$

$$C = -20,000$$

Subst for C into eqn (1)

$$F_A(t) = 20,000 - 20,000 e^{-0.03t}$$

b) The time at which 90% of the fish in the lake will become fresh is:

$$90\% = \frac{90}{100} \times 20,000 = 20,000 - 20,000 e^{-0.03t}$$

$$18,000 = 20,000 - 20,000 e^{-0.03t}$$

$$18,000 - 20,000 = -20,000 e^{-0.03t}$$

$$-2,000 = -20,000 e^{-0.03t}$$

$$0.1 = e^{-0.03t}$$

Taking \ln of both sides

$$\ln 0.1 = \ln e^{-0.03t}$$

$$-2.3026 = -0.03t$$

$$t = \frac{-2.3026}{-0.03} = 76.753 \text{ min}$$

$$t = 76.75 \text{ min}$$

Converting from minute to seconds

0.75 min - Sec

$$0.75 \times 60 = 45 \text{ seconds}$$

(c) 6 hours to minutes becomes

6 hour - 1 hour

$$6 \times 60 = 360 \text{ minutes}$$

(d) The steady-state value of the fresh amount of air in the room given = 20,000 ft³ of air

(e) The graph gave a straight line which means that the steady-state value of the amount of fresh air in the room does not change even with the expense of increase in time