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16/ENG04/056
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Step 1: Setting up a Model

Let $f_A(t)$ make the amount of fresh air in the room at time (t)

By Balance law:

$$\frac{df_A}{dt} = \text{Fresh Air inflow rate} - \text{Fresh Air outflow rate.}$$

Input of fresh Air $= 600\ ft^3/min$

Initially there was no fresh air

Hence, $f_{A(0)} = 0$

Also

Output of mixture $= 600 ft^3/min$

Mixture of fresh air & Normal Air $= 20{,}000\ ft^3/min$

$$\frac{df_A}{dt} = 600 - \frac{600}{20000} \times f_{(A)}(t)$$

$$\frac{df_A}{dt} = 600 - 0.03 f_A$$

$$\frac{df_A}{dt} = -0.03 [f_A - 20{,}000]$$

Step 2: Solution of the Model

$$\frac{df_A}{dt} = -0.03 [f_A - 20000]$$

$$\frac{df_A}{f_A - 20000} = -0.03 dt$$

Integrating both sides, we have

$$\int \frac{df_A}{f_A - 20000} = \int -0.03 dt$$

$\ln[f_A - 20{,}000] = -0.03t + c$

Taking ln of both sides

$f_A - 20{,}000 = ce^{-0.03t}$

where $c = e^c$

$f_A = 20{,}000 + ce^{-0.03t}$

Initially there was no fresh air

Hence,

$f_{A(0)} = 0$

$f_A = 20{,}000 + ce^{-0.03t}$

when $t = 0$, $f_A = 0$

Subst. for c

$f_A(t) = 20{,}000 - 20{,}000e^{-0.03t}$ [Particular Solution]

Time at which 90% of the air will 'breathe fresh

$\frac{90}{100} \times \frac{20{,}000}{1} = 20000 - 20000e^{-0.03t}$

$18000 = 20{,}000 - 20{,}000e^{-0.03t}$

$-2000 = -20000e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 76.75$ mins

Convert mins to secs.

0.75 mins $= 0.75 \times 60$ secs.

$= 45$ secs

$\therefore T = 76$ mins 45 secs.

1 hr to mins $= 60 \times 60 = 3600$ mins.

The steady-state value of the amount of fresh air in the room = [20000 ft^3 of air]

e] The steady-state value of the amount of fresh air in the room obtained from the response (graph) is given as a straight line where there is no longer increase in the amount of fresh air even though there is still increase in the time.

Hence, the amount of fresh air in the room is steady [i.e does not change] with increase in time (mins].