

AYEDMERETSE EWDRITSE N TOMERE

MECHATRONICS ENGINEERING

16/ENQ05/014

a) let $y(t)$ = amount of fresh air flowing at any time t

$$\frac{dy}{dt} = \text{inflow} - \text{outflow}$$

$$\frac{dy}{dt} = 600 - \frac{600}{20000} \text{ of } y$$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$\frac{dy}{dt} + 0.03y = 600 \implies \frac{dy}{dx} + Py + Q$$

$$P = 0.03, Q = 600$$

$$\int P dx = \int 0.03 dt = 0.03t$$

$$I.F = e^{\int P dx} = e^{0.03t}$$

$$y \cdot I.F = \int Q \cdot I.F dt$$

$$y \cdot e^{0.03t} = \int 600 \cdot e^{0.03t} dt$$

$$y \cdot e^{0.03t} = 600 \int e^{0.03t} dt$$

$$y \cdot e^{0.03t} = \frac{600}{0.03} e^{0.03t} + C$$

$$y \cdot e^{0.03t} = 20000 e^{0.03t} + C$$

$$y = \frac{20000 e^{0.03t} + C}{e^{0.03t}}$$

$$y = 20000 + \frac{C}{e^{0.03t}}$$

$$y = 20000 + C e^{-0.03t}$$

$$y(0) = 0$$

$$0 = 20000 + C e^{-0.03 \times 0}$$

$$0 = 20000 + C$$

$$C = -20000$$

$$\therefore y = 20000 - 20000 e^{-0.03t}$$

b 90% of 20,000

$$= \frac{90}{100} \times 20000 = 18000$$

$$18000 = 20000 - 20000 e^{-0.03t}$$

$$20000 e^{-0.03t} = 2000$$

$$e^{-0.03t} = \frac{2000}{20000}$$

$$e^{0.03t} = 0.1$$

$$-0.03t = \ln 0.1$$

$$t = \frac{\ln 0.1}{-0.03}$$

$$= 76.7$$

$$= 77 \text{ min}$$

d The steady state value is approximately 20000

e The rate of change of fresh air with time is approximately constant

$$y=20000-20000*\exp(-0.03*t)$$

t	y
0	0
5	2785.84
10	5183.636
15	7247.437
20	9023.767
25	10552.67
30	11868.61
35	13001.25
40	13976.12
45	14815.19
50	15537.4
55	16159
60	16694.02
65	17154.52
70	17550.87
75	17892.02
80	18185.64
85	18438.37
90	18655.89
95	18843.11
100	19004.26
105	19142.96
110	19262.34
115	19365.09
120	19453.53
125	19529.65
130	19595.16
135	19651.55
140	19700.09
145	19741.86
150	19777.82
155	19808.77
160	19835.41
165	19858.33
170	19878.07
175	19895.05
180	19909.67
185	19922.25
190	19933.08
195	19942.4
200	19950.42
205	19957.33
210	19963.27
215	19968.39

220	19972.79
225	19976.58
230	19979.84
235	19982.65
240	19985.07
245	19987.15
250	19988.94
255	19990.48
260	19991.81
265	19992.95
270	19993.93
275	19994.77
280	19995.5
285	19996.13
290	19996.67
295	19997.13
300	19997.53
305	19997.88
310	19998.17
315	19998.43
320	19998.65
325	19998.83
330	19999
335	19999.14
340	19999.26
345	19999.36
350	19999.45
355	19999.53
360	19999.59

