Ambrose Victor
16/Eng06/011
Mechanical Eng
Eng 387

1 It is discovered that 600 ft^3/min of fresh air flows into a room containing 20000 ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of 600 cubic feet per minute (cfm). If the room contains no fresh air initially

Sol (develop a model for the amount of fresh air at any time)

Let y represent fresh air by

Rate of Accumulation = rate of inflow - rate of outflow

$$\frac{dy}{dt} = 600 - 0.03y$$

rate of inflow = $600\ ft^3/min$

rate of outflow = $\frac{600}{20000} \times y = 0.03y$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{dy}{(y-20000)} = -0.03\,dt$$

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$y - 20000 = y_e e^{-0.03t}$$

$$y = 20000 + y_0 e^{-0.03t}$$

if the room contained no fresh air

$y = 0$, $t = 0$

$$0 = 20000 + y_0$$

$$y_0 = -20000$$

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000 (1 - e^{-0.03t})$$

This is the model for the amount of air at any time t

b. Calculate the time which 90% of the air in the room will have become fresh

Sol

Room contains $20000 ft^3$ of air

90% of 20000

$= 18000 ft^3/min$

If from the model

$$y = 20000 (1 - e^{-0.03t})$$

$=$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$e^{-0.03t} = -\left(\frac{18000}{20000} - 1\right)$$

$$e^{-0.03t} = +0.1$$

$$-0.03t = \ln(0.1)$$

$$-0.03t = -2.303$$

$$t = \frac{2.303}{0.03}$$

$$= 76.77 min.$$

d) From the dynamic response plotted, the steady-state value of the amount of fresh air in the room is 20 000 ft^3 of air

e) It was noticed that the value of amount of fresh air steadily increase until it got to 20,000ft^3 of air. Therefore despite the increase in time the amount of fresh air remained 20,000ft^3 giving the steady-state value. In conclusion, 20000ft^3 of air is the maximum air for the room.