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16/ENG01/005

Chemical Engineering

ENG 282 - Assignment 2

Solution

Let $y(t)$ denote the amount of fresh air present at time t.

y' = fresh air inflow rate - fresh air outflow rate.

⇒ The room contains $20000\,ft^3$ of air

outflow rate = $600\,ft^3/min$.

$$\Rightarrow \frac{600}{20000} = 0.03$$

This means that $0.03\,ft^3$ of the total air flows out, per minute.

y' = inflow rate = $600\,ft^3/min$

$$\Rightarrow y' = 600 - 0.03y$$

$$y' = -0.03(y - 20000)$$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\int \frac{dy}{y - 20000} = \int -0.03\,dt$$

$$\Rightarrow \ln|y - 20000| = -0.03t + c$$

$$y - 20000 = e^{-0.03t + c}$$

$$y - 20000 = e^{-0.03t} \cdot e^{c} \quad \text{let } e^{c} = a$$

$$y - 20000 = ae^{-0.03t}$$

$$y(t) = 20000 + ae^{-0.03t}$$

Initially, there is no fresh air in the room,

$$y(t) = 0$$

$$0 = 20000 + ae^{-0.03 \times 0}$$

$$0 = 20000 + a \Rightarrow a = -20000$$

$$y(t) = 20000 - 20000e^{-0.03t}$$

b) 90% of the air in the room;

$\frac{90}{100} \times 20000$

$= 18000$

$\therefore y(t) = 18000$

$y(t) = 20000 - 20000\,e^{-0.03t}$

$18000 = 20000 - 20000\,e^{-0.03t}$

$20000\,e^{-0.03t} = 20000 - 18000$

$e^{-0.03t} = \frac{2000}{20000} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = 76.75$ minutes

$t \approx 77$ minutes

(d) From the graph, steady-state value $\approx 20000\,ft^3$

(e) It is at this point, that an increase in time will not lead to an increase in amount of fresh air in the room.