

MAC-ETELI GOLDEN

16/ENG05/021

MECHATRONICS

4

1a

Rate of inflow = $600 \text{ ft}^3/\text{min}$

Rate of outflow = $\frac{600}{20000} \times y = 0.03y$

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$y - 20000 = y_0 e^{-0.03t}$$

$$y = 20000 + y_0 e^{-0.03t}$$

When the room contained no fresh air

$$y = 0, t = 0$$

$$0 = 20000 + y_0$$

$$y_0 = -20000$$

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t})$$

1b

Room is containing 20000 ft^3 of air
90 % of $20000 = 18000 \text{ ft}^3/\text{min}$
from the model

$$y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$e^{-0.03t} = \left[\frac{18000}{20000} - 1 \right]$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$-0.03t = -2.303$$

$$t = \frac{2.303}{0.03}$$

$$t = 76.77 \text{ min}$$

D) The steady-state value of the amount of air in the room is approximately $20,000 \text{ ft}^3/\text{min}$

E) If the value of the amount of fresh air in the room increases and remained constant at 20,000 ft³/min