

QUESTION 1

Step 1: Using the balance law, let  $F_A(t)$  denote the amount of fresh air in the room at time  $t$ .

Using the balance law:  
 $\frac{dF_A}{dt} = \text{Fresh Air Inflow rate} - \text{Fresh Air outflow rate}$   
 Fresh air inflow rate =  $600 \text{ ft}^3/\text{min}$   
 Initially, there was no fresh air, hence  $F_A(0) = 0$ .

Also, Mixture output =  $600 \text{ ft}^3/\text{min}$   
 Mixture of fresh air and Normal Air =  $20000 \text{ ft}^3$  fresh

Mixture of fresh air and Normal Air =  $20000 \text{ ft}^3$  fresh  
 $\therefore \frac{dF_A}{dt} = 600 - 0.03 F_A$   
 $\frac{dF_A}{dt} = -0.03 (F_A - 20000)$

Step 2: Solution of the model.

$\frac{dF_A}{dt} = -0.03 (F_A - 20000)$   
 $\frac{dF_A}{F_A - 20000} = -0.03 dt$

$\int \frac{dF_A}{F_A - 20000} = \int -0.03 dt$

$\ln (F_A - 20000) = -0.03t + C$

Dividing by  $\ln$  of both sides  
 $F_A - 20000 = Ce^{-0.03t}$

where  $C = e^C$   
 $F_A = 20000 + Ce^{-0.03t}$

Initially there was no fresh air, Hence,

$F_A(0) = 0$   
 $F_A = 20000 + Ce^{-0.03t}$

$C = -20000$   
 $\therefore F_A(t) = 20000 - 20000e^{-0.03t}$

The above is the particular solution

(b) Time at which 90% of the air will become fresh.

$90 \times 20000 = 20000 - 20000e^{-0.03t}$   
 $18000 = 20000 - 20000e^{-0.03t}$   
 $0.12 e^{-0.03t} = 1$   
 $\therefore t = 0.12 - 0.03t$

$t = 76.75 \text{ mins}$   
 $60 \text{ mins} \rightarrow 1 \text{ minute}$   
 $\therefore t = 76.75 \text{ mins} \approx 0.75 \times 60 = 45 \text{ secs}$   
 $\therefore t = 76 \text{ mins } 45 \text{ seconds}$

(c) Hours to minutes =  $6 \times 60 = 360 \text{ minutes}$

(d) The steady-state value of the amount of fresh air in the room =  $20000 \text{ (ft}^3 \text{ of air)}$ .

(e) The steady-state value of the amount of fresh air to the room obtained from the graph (response) is given to be a straight line where there is no longer increase in the amount of fresh air even though there is still increase in the time.

Hence, the amount of fresh air in the room is steady with increase in time (min).

$$F(t) = 20000 - 20000 \cdot \exp(-0.03t)$$

20000  
-0.03

t	F(t)
0	0
5	2785.84047
10	5183.63559
15	7247.43697
20	9023.76728
25	10552.6689
30	11868.6068
35	13001.245
40	13976.1158
45	14815.1948
50	15537.3968
55	16159.0018
60	16694.0222
65	17154.5186
70	17550.8714
75	17892.0155
80	18185.6409
85	18438.3667
90	18655.8897
95	18843.1136
100	19004.2586
105	19142.9575
110	19262.3367
115	19365.0873
120	19453.5256
125	19529.6451
130	19595.1618
135	19651.5525
140	19700.0885
145	19741.8637
150	19777.8201
155	19808.768
160	19835.4051
165	19858.3318
170	19878.0651
175	19895.0496
180	19909.6684
185	19922.2509
190	19933.0807
195	19942.402
200	19950.425



