NAME: OKPOKPARORO MARO
COURSE: ENG 282
DEPT : MECHANICAL ENGINEERING
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Assignment 2

Solution

(a) Setting up the model

Let $F_A(t)$ be the amount of fresh air in the room at Time t.

Using Balance Law,

$\frac{dF_A}{dt}$ = Fresh Air Inflow rate − Fresh Air outflow rate

Input of fresh Air = $600 ft^3/min$

Initial fresh air rate = $0 = F_A$

output mixture = $600 ft^3/min$

Fresh Air and Normal Air mixture = $20,000 ft^3/min$

$$= \frac{dF_A}{dt} = 600 - \frac{600}{20,000} F_A(t)$$

$$= \frac{dF_A}{dt} = 600 - 0.03 F_A$$

$$= \frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

ii) Solution to the model

$$\frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

$$dF_A = -0.03 dt$$

$= \frac{dF_A}{F_A - 20,000} = -0.03dt$

Integrate both sides,

$\int \frac{dF_A}{F_A - 20,000} = \int -0.03\,dt$

$\ln(F_A - 20,000) = -0.03t + C$

Take ln of both sides

$F_A - 20,000 = C \cdot e^{-0.03t}$

where $e^C = C$

$\therefore F_A = 20000 + Ce^{-0.03t}$

F_A = general solution.

Recall

$F_A = 20,000 + Ce^{-0.03t}$

where $t = 0$, $F_A = 0$

$0 = 20,000 + C \cdot e^{-0.03(0)}$

$C = -20,000$

Therefore, Substitute the value for C

$F_A(t) = 20,000 - 20,000e^{-0.03t}$

$F_A(t)$ = particular solution

(b) The time at which 90% of the air in the room will become fresh is,

$90\% = \frac{90}{100} \times 20,000 = 20,000 \cdot e^{-0.03t}$

$= 18000 = 20,000 - 20,000e^{-0.03t}$

$18000 - 20,000 = -20,000e^{-0.03t}$

$-2,000 = -20,000e^{-0.03t}$

$0.1 = 1 \cdot e^{-0.03t}$

take ln of LHS to eliminate e,

$\ln 0.1 = -0.03t$

$-2.3026 = -0.03t$

$t = \frac{-2.3026}{-0.03}$

$t = 76.753$ min

$t \approx 76.75$ mins

convert .75 min - sec

$= 0.75 \times 60 = 45$s

$t =$ 76 minutes 45 seconds

(c) 6 hours to minutes becomes,

60 mins - 1 hour

6 hours $= 6 \times 60 = 360$ minutes

(d) The steady-state value of the fresh amount of air in the room given $= 20{,}000\ ft^3$ of air

(e) The graph gave a corresponding straight line which entails that the steady-state value of the amount of fresh air in the room does not change even with the expense of increase in time,