Assignment 2

Solution

Step 1: Setting Up a Model

Let $F_A(t)$ denote the amount of fresh air in the room at time (t)

By Balance law

$\frac{\partial f_A}{\partial t}$ = fresh air in flow rate − fresh air out flow rate

Input of fresh air = $600\ ft^3/min$

Initially there was no fresh air

Hence, $f_A(0) = 0$

Also

Output of Mixture = $600 ft^3/mm$

Mixture of fresh air and normal air = 20,000

$\frac{\partial f_A}{\partial t} = 600 - \frac{600}{20,000} \times F_A(t)$

$\frac{\partial f_A}{\partial t} = 600 - 0.03 f_A$

$\frac{\partial f_A}{\partial t} = -0.03 (f_A - 20,000)$

Step 2: Solution of the Model

$\frac{\partial f_A}{\partial t} = -0.03 (f_A - 20,000)$

$\frac{\partial f_A}{f_A - 20,000} = -0.03 \partial t$

Integrating both sides we have

$\int \frac{df_A}{f_A - 20,000} = \int -0.03 dt$

$\ln (f_A - 20,000) = -0.03t + C$

Taking (exp) of both sides

$f_A - 20,000 = Ce^{-0.03t}$

Where $C = e^C$

$F_A = 20,000 + Ce^{-0.03t}$

Initially there was no fresh air

Hence

$f_A(0) = 0$

$F_A = 20{,}000 + ce^{-0.03t}$

Where $t = 0$ for 0

$0 = 20{,}000 + c \cdot e^{-0.03(0)}$

$c = -20{,}000$

Substitute for 0

$F_A(t) = 20{,}000 - 20{,}000e^{-0.03t}$

(Particular Solution)

(b) - Time at which 90% of the 0 will become fresh

$\frac{90}{100} \times \frac{20{,}000}{1} = 20{,}000 - 20{,}000e^{-0.03t}$

$18{,}000 = 20{,}000 - 20{,}000e^{-0.03t}$

$-2000 = -20{,}000e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 16.15$ Mins

Convert mins to seconds

$= 15$ Seconds

$T = 16.15$ Mins : ~~15 Seconds~~

(c) 6 hours to Minutes $= 60 \times 60 = 36{,}000$ mins

(d) The Steady State Value of the Amount of fresh air in the room $= 20{,}000$ (ft^3 of air)

(e) The Steady-State Value of the Amount of fresh air in the room obtained from the response Graph is given as a straight line where there is no longer increase in the amount of fresh air even though there is still increase in the time.

Hence, the amount of fresh air in the room is steady, it does not change with increase in time (minutes).