Q1 Setting up the model
let $f_A(t)$ be the amount of fresh air in the room at time t.
using Balance law;

$$\frac{df_x}{dt} = \text{fresh Air inflow rate} - \text{fresh Air outflow rate}$$

Input of fresh Air = $600ft^3/mins$
Initial Fresh air $= 0 = f_x$
Output mixture $= 600ft^3/min$
fresh Air and Normal Air mixture $= 20,000ft^3/min$

$$= \frac{df_A}{dt} = 600 - \frac{600}{20,000} f_A(t)$$

$$= \frac{df_x}{dt} = 600 - 0.03f_x$$

$$= \frac{df_x}{dt} = -0.03(f_A - 20,000)$$

(ii) Solution to the model.

$$\frac{df_A}{dt} = -0.03(f_X - 20,000)$$

$$\frac{df_x}{dt} = -0.03dt$$

$$= \frac{df_A}{F_A - 20,000} = -0.03dt$$

Integrate both sides

$$\int \frac{df_A}{f_x - 20,000} = \int -0.03dt$$

$$\ln(f_A - 20,000) = -0.03t + C$$

Take ln of both side;

$Fx - 20{,}000 = c\, e^{-0.03t}$

where $e^c = c$

$\therefore Fx = 20{,}000 + c\, e^{-0.03t}$

fx = general solution

Recall

$Fx = 20{,}000 + ce^{-0.03t}$

where $t = 0$; $fx = 0$

$0 = 20{,}000 + c \cdot e^{-0.03(0)}$

$c = -20{,}000$

Therefore, substitute the value for c

$Fx(t) = 20{,}000 - 20{,}000e^{-0.03t}$

$Fx(t)$ = particular solution

b. The time at which 90% of the air in the room will become fresh is ;

$90\% = \frac{90}{100} \times 20{,}000 = 20{,}000\, e^{-0.03t}$

$= 18000 = 20{,}000 - 20{,}000e^{-0.03t}$

$= 18000 - 20{,}000 = 20{,}000e^{-0.03t}$

$-20{,}000 = -20{,}000\, e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 76.75$ mins

Convert mins to seconds

0.75 mins = 0.75 × 60 seconds

= 45 second

$\therefore$ T = 76 minutes 45 seconds

c. 6 hours to minutes = 60 × 60 = 360 minutes

d. The steady-state value of the amount of fresh air in the room = 20,000 (ft^3 of air)