Solution

(a) Setting up the model

Let $f_A(t)$ be the amount of fresh air in the room at Time t.

Using Balance Law,

$\frac{dF_A}{dt}$ = Fresh air inflow rate - Fresh air outflow rate

Input of fresh Air = $600 ft^3/min$

Initial fresh air rate = $0 = f_A$

Output mixture = $600 ft^3/min$

Fresh Air and Normal Air mixture = $20,000 ft^3/min$

$$\frac{dF_A}{dt} = 600 - \frac{600}{20,000} F_A(t)$$

$$\frac{dF_A}{dt} = 600 - 0.03 F_A$$

$$\frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

ii) Solution to the model

$$\frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

$$\frac{dF_A}{F_A - 20,000} = -0.03 dt$$

Integrate both sides,

$$\int \frac{dF_A}{F_A - 20,000} = \int -0.03\,dt$$

$$\ln(F_A - 20,000) = -0.03t + C$$

Take ln of both sides

$$F_A - 20,000 = C \cdot e^{-0.03t}$$

where $e^C = C$

$$\therefore F_A = 20000 + Ce^{-0.03t}$$

F_A = general solution

Recall

$$F_A = 20,000 + Ce^{-0.03t}$$

where $t = 0$, $F_A = 0$

$$0 = 20,000 + C \cdot e^{-0.03(0)}$$

$$C = -20,000$$

Therefore, Substitute the values for C

$$F_A(t) = 20,000 - 20,000\,e^{-0.03t}$$

$F_A(t)$ = particular solution

(b) The time at which 90% of the air in the room will become fresh is,

$$90\% = \frac{90}{100} \times 20,000 = 20,000\,e^{-0.03t}$$

$$= 18000 = 20,000 - 20,000\,e^{-0.03t}$$

$$18000 - 20,000 = -20,000e^{-0.03t}$$

$$-2,000 = -20,000\,e^{-0.03t}$$

$0.1 = 1 \cdot e^{-0.03t}$

take ln of LHS to eliminate e,

$\ln 0.1 = -0.03t$

$-2.3026 = -0.03t$

$t = \frac{-2.3026}{-0.03}$

$t = 76.753$ mins

1 1 76.75mins

convert .75 min – sec

$= 0.75 \times 60 = 45s$

$t = 76$ minutes 45 seconds

(b) 6 hours to minutes becomes,

60mins – 1 hour

6hours $= 6 \times 60 = 360$ minutes

(a) The steady-state value of the fresh amount of air in the room gives $= 20{,}000\ ft^3$ of air

(c) The graph gave a corresponding straight line which entails that the steady-stated value of the amount of fresh air in the room does not change even with the expense of increase in time.