Mechanical Engineering

ENG 282

It is discovered that 600 ft^3/min of fresh air flows into a room containing 20000 ft^3 of air. The mixture which is practically uniform by circulating fans, is exhausted at a rate of 600 ft^3/min. If the room contains no fresh air initially

(a)

Develop a model for the amount of fresh air at any time t in the room

step 1: Setting up a model

$A(t)$ = amount of air in cubic feet in the room at time t

$\frac{dA}{dt}$ = rate of increase of fresh air/unit time (mca)

$\frac{dA}{dt}$ = Fresh air in − fresh air out

fresh air in = 600 ft^3/min

Air out = $\frac{600}{20000}$ of A = $\frac{600A}{20000}$

$\frac{dA}{dt} = 600 - 0.03A$ separating variables

$\frac{dA}{1-0.00000SA} =$

$\frac{dA}{dt} = 600 - \frac{600A}{20000}$

$\frac{dA}{dt} = \frac{600(20000) - 600A}{20000} = \frac{600}{20000}(20000 - A)$

$\frac{dA}{dt} = \frac{600}{20000}(20000 - A)$ separating variables

$\frac{dA}{} = \frac{600}{} dt$

Integrating both sides

$$\int \frac{dA}{20000-A} = \frac{600}{20000} \int dt$$

$$-\ln(20000-A) = \frac{600}{20000} t + C$$

$$-\ln(20000-A) = 0.03t + C$$

multiply both sides by -1

$$\ln(20000-A) = -0.03t - C$$

$$20000-A = e^{-0.03t} \cdot e^{-C} = e^{-0.03t-C}$$

$$A = 20000 - e^{(-0.03t-C)}$$

$t=0$, $A(t) = A(0) = 20000 = e^{-C} = A_0$

$$A = 20000 - 20000\, e^{-0.03t}$$ General solution

(b)

Calculate the time at which 90% of the air in the room will become fresh

90% of $20000\,ft^3 = 18000\,ft^3$

$$18000 = 20000 - 20000\, e^{-0.03t}$$

$$18000 = 20000(1 - e^{-0.03t})$$

divide both sides by 20000

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9 = 0.1$$

$$e^{-0.03t} = 0.1$$ taking ln of both sides

$$-0.03t = \ln(0.1)$$

$$-0.03t = -2.3$$ divide both sides by -0.03

$$t = \frac{-2.3}{-0.03} = 76.75 \text{ min}$$

$t \approx 77$ min = time taken which 90% of air in the room

$x_{min} = 6hr$

$$x_{min} = \frac{6\,hr\,(60\,min)}{1\,hr} = 360\,min$$

using excel plot a graph for $t = 0$ to $t = 360\,min$ steptime of 5min

d

$$\lim_{t \to \infty} \left(20000 - 20000\, e^{-0.03t}\right)$$

$$= \lim_{t \to \infty} 20000 - \lim_{t \to \infty} \left(\cancel{20000 \cdot e^{-0.03t}}^{\to 0}\right)$$

$$= 20000$$

e

as the time approaches infinity