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COMPUTER ENGR.
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STEP 1: Setting up a model

let $F_A(t)$ denote the amount of fresh air in the room at time t

By balance law:

$$\frac{dF_A}{dt} = \text{Fresh air in flow rate} - \text{Fresh air out flow rate.}$$

Input of fresh air = $600 \text{ ft}^3/\text{min}$

initially there was no fresh air

Hence, $F_A(0) = 0$

Also

Output of mixture = $600 \text{ ft}^3/\text{min}$

Mixture of fresh air and Normal air
= $20,000 \text{ ft}^3/\text{min}$

$$\frac{dF_A}{dt} = 600 - \frac{600}{20000} \times F_A(t)$$

$$\frac{dF_A}{dt} = 600 - 0.03 F_A$$

$$\frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

STEP 2: SOLUTION OF THE MODEL

$$\frac{dF_A}{dt} = -0.03 (F_A - 20,000)$$

$$\frac{dF_A}{F_A - 20,000} = -0.03 dt$$

$$F_A - 20,000$$

Integrating both sides, we have

$$\int \frac{dF_A}{F_A - 20,000} = \int -0.03 dt$$

$$\ln(F_A - 20,000) = -0.03t + C$$

Taking (exp) of both sides

$$F_A - 20,000 = C e^{-0.03t}$$

where $C = e$

$$F_A = 20,000 + C e^{-0.03t}$$

Initially there was no fresh air
Hence,

$$F_A(0) = 0$$

$$F_A = 20,000 + C e^{-0.03t}$$

where $t = 0, F_A = 0$

$$0 = 20,000 + C e^{-0.03(0)}$$

$$C = -20,000$$

Substituting for C

$$F_A(t) = 20,000 - 20,000 e^{-0.03t}$$

(particular solution)

(b) Time at which 90% of the air will become fresh

$$\frac{90}{100} \times \frac{20,000}{1} = 20,000 - 20,000 e^{-0.03t}$$

$$18,000 = 20,000 - 20,000 e^{-0.03t}$$

$$-2,000 = -20,000 e^{-0.03t}$$

$$0.1 = e^{-0.03t}$$

$$\ln 0.1 = -0.03t$$

$$t = 76.75 \text{ mins}$$

Convert mins to second

= 45 seconds

$\therefore T = 76 \text{ mins } 45 \text{ seconds}$

c) 6 hours to minutes = $6 \times 60 = 360 \text{ mins}$.

d) The steady state value of the amount of fresh air in the room = $20,000 \text{ (ft}^3 \text{ of air)}$

e) The steady-state value of the amount of fresh air in the room obtained from the response (graph) is given as straight line where there is no longer increase in the amount of fresh air even though there is still increase in the time.

Hence, the amount of fresh air in the room is steady (it does not change) with increase in time (minutes).