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(a) Setting up a model
Let $f_A(t)$ denote the amount of fresh air in a
at time (t)
By balance law
$\frac{df_A}{dt}$: Fresh air inflow rate - fresh air outflow ra
dt

Input of fresh air = 600ft³/min
Initially there was no fresh air
Hence $f_A(0) = 0$
Also mixture
Output of = 600ft³/min
Mixture of fresh air and normal = 20000ft³

$\frac{df_A}{dt} = 600 - \frac{600}{20000} \times f_A(t)$

$\frac{df_A}{dt} = 600 - 0.03 f_A$

$\frac{df_A}{dt} = -0.03(f_A - 20000)$

Solution of the model

$$\frac{df_A}{dt} = -0.03(f_A - 20000)$$

$$\frac{df_A}{f_A - 20000} = -0.03dt$$

Integrating both sides we have -

$$\int \frac{df_A}{f_A - 20000} = \int -0.03dt$$

$$\ln(f_A - 20000) = -0.03t + c$$

Taking exp of both sides

$$f_A - 20000 = Ce^{-0.03t}$$

where $C = e$

$f_A = 20000 + Ce^{-0.03t}$

where $t = 0$, $f_A = 0$

$0 = 20000 + Cxe^{-0.03(0)}$

$C = -20000$

Substitute for C

$f_A(t) = 20000 - 20000e^{-0.03t}$ (particular soln)

b Time at which 90% of the air will become fresh

$\frac{90}{100} \times 20000 = 20000 - 20000e^{-0.03t}$

$18000 = 20000 - 20000e^{-0.03t}$

$-2000 = -20000e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 76.75$ mins

c 6 hours 60 minutes $= 60 \times 60 = 360$ minutes

d The steady state value at the amount of fresh air in the room $= 20000$ ft^3 of air

c 6 hours 60 minutes : 60 × 60 = 360 minutes

d The steady state value at the amount of fresh air in the room = 20000 ft^3 of air

e The steady-state value of the amount of fresh air in the room obtained from the response (graph) is given as straight line where there is no longer increase on the amount of fresh air even though there is still increase in the time. Hence the amount of fresh air in the room is steady with increase in time (minutes)