Name: Aliyu mohammed
Matric Number: 1615WG04/009
Department: Elect / Elect

Step 1 set up a model

Let $f_A(t)$ denote the amount of fresh air in the room at time (t)

using the balance law

$\frac{df_A}{dt}$ = fresh Air inflow rate - fresh air out flow rate

fresh air inflow rate = $600 ft^2/min$

Initially there was no fresh air

Hence, $f_A(0) = 0$

Also

mixture output = $600 ft^3/min$

mixture of fresh air and Normal Air = $20000 ft^2/min$

$$\frac{df_2}{dt} = 600 - \frac{600}{20000} \times f_A(t)$$

$$\frac{df_2}{dt} = 600 - 0.03 f_2$$

$$\frac{df_2}{dt} = -0.03(f_2 - 20,000)$$

Step 2:- solution of the model.

$$\frac{df_2}{dt} = 0.03(f_2 - 20,000)$$

$$\frac{df_2}{f_A - 20000} = -0.03 dt$$

$$\int \frac{df_2}{f_A - 20000} = -0.03 \int dt$$

$\ln(f_A - 20000) = -0.03t + c$

$f_2 - 20000 = Ce^{-0.03t}$

where $C = e^c$

$f_A = 20000 + Ce^{-0.03t}$

Initially there was no fresh air

Hence,

$f_A(t) = 0$

$f_A = 20000 + Ce^{-0.036}$

when $t = 0$, $f_A = 0$

$\therefore 0 = 20{,}000 + C \times e^{-0.03(0)}$

$C = -20000$

$\therefore f_2(t) = 20000 - 2000e^{-0.03t}$

The above is the particular solution

Time at when 90% of the air will become fresh

$\frac{90}{100} \times \frac{20000}{1} = 20000 - 20000e^{-0.03t}$

$18000 = 20000 - 20000e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 76.75$ mins

mins → seconds

60 sec → 1 mints

$0.75 \times 60 = 45$ sec

i.e. $t = 76$ mins 45 Seconds

6 hours to minutes $= 6 \times 60 = 360$ minutes

(d) The steady state value of the amount of fresh air in the room $= 20{,}000$ (ft^3 of air)

(e) The steady state value of the amount of fresh air to the room obtained from the graph (response) to given for be a straight line where there is no longer increase in the amount of fresh air even though there is still ~~the~~ increase in the time

Here, the amount of fresh air in the room is steady with increase in time (mins)