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Step 1: Set up a model

Let F(t) denote the amount of the fresh air in the room of time(t).

Using Balance law.

dF/dt = Fresh Air In flow rate − fresh air out flow rate

Fresh air in flowrate = 600ft³/min.

Initially there was no fresh air

Hence F(0) = 0.

Also

Mixture Output = 600ft³/min

Mixture of fresh air and Normal Air = 20000ft³/min

$$\frac{dF_A}{dt} = 600 - \frac{600}{20000} \times F(t)$$

$$\frac{dF_A}{dF_t} = 600 - 0.03F_A$$

$$\frac{dF_A}{dt} = -0.03(F_A - 20{,}000)$$

Step 2: Solution to the model

$$\frac{dF_A}{dt} = -0.03(F_A - 20{,}000)$$

$$\frac{dF_A}{F_A - 20000} = -0.03\delta t$$

$$\int \frac{dF_A}{F_A - 20000} = -0.003 \int dt.$$

$$\ln(F_A - 20000) = 0.03.$$

$$F_A - 20000 = Ce^{-0.03t}$$

When $t = 0$, $F_n = 0$.

$\therefore 0 = 20{,}000 + C \times e^{-0.03(0)}$

$C = -20000$

$F(t) = 20000 - 20000 e^{-0.03t}$

The above is the particular solution

b) Time at which 90% of the air will become fresh

$\frac{90}{100} \times 20000 = 20000 - 20000 e^{-0.03t}$

$18{,}000 = 20000 - 20000 e^{-0.03t}$

$0.1 = e^{-0.03t}$

$\ln 0.1 = -0.03t$

$t = 76.75 \text{ mins}$

But $60 \text{ sec} = 1 \text{ m}$

$0.75 \times 60 = 45 \text{ sec}$

$t = 76 \text{ mins}, 45 \text{ sec}$

c) 6 hrs to minutes $= 6 \times 60 = 360 \text{ mins}$

d) The steady state value of the amount of fresh air in the room $= 20{,}000$ (ft^3 of air)

e) The steady state value of the amount of fresh air to the room obtained from the graph (response is given to be a straight line where it no longer increase in the amount of fresh air even though there is still increase in the time.

Here, the amount of fresh air in the room is steady with increase in time (mins)