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16/ENG 07/018.
PETROLEUM ENGINEERING
ENG 282
ASSIGNMENT III
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Question 1

i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpreting the result in physical or other terms.

ii)

a) Using the balance Law (Law of Conservation of Mass)
b) Forming a differential equation from an existing algebraic equation of the system.

Question 2.

$$r = (t^2+3t)i - 2\sin 3t j + 3e^{2t}k,$$

determine

i) $$\frac{dr}{dt} = i\frac{da_x(t)}{dt} + j\frac{da_y(t)}{dt} + k\frac{da_z(t)}{dt}$$

$$\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t}k$$

ii) $$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$$

at $t = 0$

iii) $\frac{d^2r}{dt^2} = 2i + 0j + 12k = 2i + 12k$

at $t = 0$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2i)^2 + (12k)^2}$$

$i \cdot i = j \cdot j = k \cdot k = 1$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{148} = \sqrt{4 \times 37} = 2\sqrt{37}$$

$$\therefore \left|\frac{d^2r}{dt^2}\right| = 2\sqrt{37} \simeq 12.17$$

<u>Question 3</u>

$A = x^2yi + (xy + yz)j + xz^2k$,
$B = yzi - 3xzj + 2xyk$, and
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$,
at point $(1, 2, 1)$

i) $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\nabla\phi = (6xy + yz - 0 - 0)i + (3x^2 + xz - 8yz^2 - 0)j + (0 + xy - 8y^2z - 0)k$

at point $(1, 2, 1)$

$\nabla\phi = (12 + 2)i + (3 + 1 - 16)j + (2 - 32)k$

$\nabla\phi = 14i - 12j - 30k$

$\nabla\phi = 2(7i - 6j - 15k)$ ans

ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = x^2yi + (xy + yz)j + xz^2k$

$a_x = x^2 y$
$a_y = (xy + yz)$
$a_z = xz^2$

$\nabla \cdot A = 2xy + (x + \cancel{z}) + 2xz$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

iii) ~~$\nabla \times B$~~ at point $(1,2,1)$

$\nabla \cdot A = 4 + (1+1) + \cancel{2} = 4 + 2 + 2$
$\nabla \cdot A = \cancel{8} = \underline{8}$

$\therefore \nabla \cdot A = \underline{8}$

iii) $\nabla \times B$

$B = yzi - 3xzj + 2xyk$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]$$
$$+ k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$$

$$= i[2x + 3x] - j[2y - y] + k[-3z - z]$$

$= 5xi - yj - 4zk$
at point $(1,2,1)$

$= 5i - 2j - 4k$

$\therefore \nabla \times B = \underline{5i - 2j - 4k}$

iv) grad. div $A = \nabla\cdot(\nabla A) = \nabla^2 A$

grad. div $A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y} + \frac{\partial^2 A}{\partial z}$

$A = x^2 y i + (xy + yz) j + xz^2 k$

div. $A = 2xy + (x+z) + 2zx$

grad. div $A = i(2y + 1 + 2z) + (2x)j + (1+2x)k$

at point $(1,2,1)$

$= i(4+1+2) + (2)j + (1+2)k$

grad. div $A = 7i + 2j + 3k$

v) Curl Curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2 y)\right] + k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2 y)\right]$$

$= i[0 - y] - j[z^2 - 0] + k[y - x^2]$

$= -yi - z^2 j + [y - x^2]k$

Curl $A = -yi - z^2 j + [y - x^2]k$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y - x^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right] + k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$$

$= i[1 + 2z] - j[-2x] + k[1]$

Curl Curl $A = [1+2z]\hat{i} + 2xj + K$

at Point $(1, 2, 1)$

Curl Curl $A = \underline{\underline{3i + 2j + K}}$ ans