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16/ENG07/011
PETROLEUM ENGINEERING
ENG 282
Assignment - III
05/04/18

1)

i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpreting the result in physical or in order terms.

ii) a) Using Balance Law - Law of conservation of mass.

b) Forming a differential equation from an existing algebraic equation of the system.

2)

$r = (t^2+3t)i - 2\sin 3t j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0) j + 12e^{2(0)}k$

$= 2i + 12k$

iv) $\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2i)^2 + (12k)^2}$

$i \cdot i = j \cdot j = k \cdot k = 1$

$= \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37}$

$\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 12.17$

3) at point $\overset{x}{(1}, \overset{y}{2}, \overset{z}{1})$

$A = x^2y\,i + (2y + yz)\,j + xz^2k$

$B = yz\,i - 3xz\,j + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\nabla\phi = (6xy + yz)\,i + (3x^2 + xz - 8yz^2)\,j + (xy - 8y^2z)\,k$

at point $(1, 2, 1)$

$\nabla\phi = (6(1)(2) + (2)(1))\,i + ((3(1)^2 + (1)(1) - 8(2)(1)^2)\,j + [(1)(2) - 8(2)^2(1)]\,k$

$\nabla\phi = 14i - 12j - 30k$

$\nabla\phi = 3(14/3\,i - 4j - 10k)$

ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$
at $(1,2,1)$

$\nabla \cdot A = 4+2+2$
$\nabla \cdot A = 8$

ii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$

$i\left[\partial/\partial y(2xy) - \partial/\partial z(-3xz)\right] - j\left[\partial/\partial x(2xy) - \partial/\partial z(yz)\right]$

$+ k\left[\partial/\partial x(-3xz) - \partial/\partial y(yz)\right]$

$\nabla \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

$\nabla \times B = 5x\,i - y\,j - 4z\,k$
at point $(1,2,1)$
$\nabla \times B = 5(1)\,i - (2)\,j - 4(1)\,k$
$\nabla \times B = 5i - 2j - 4k$

iv) $grad \cdot div A = \nabla \cdot (\nabla A) - \nabla^2 A$

$grad\ div A = \partial/\partial x \frac{\partial A}{\partial x} + \partial/\partial y \frac{\partial A}{\partial y} + \frac{\partial}{\partial z} \frac{\partial A}{\partial z}$

$A = x^2 y i + (xy + yz) j + xz^2 k$

$div A = 2xy + (x + z) + 2xz$

$grad \cdot div A = \partial/\partial x \frac{\partial A}{\partial x} i + \partial/\partial y\ \partial A/\partial y\ j + \partial/\partial z\ \partial A/\partial z\ k$

$grad\ div A = (2y + 1 + 2z) i + (2x) j + (1 + 2x) k$

$grad \cdot div A$ at point $(1,2,1)$ $= (2(2) + 1 + 2(1)) i + (2(1)) j + (1 + 2(1)) k$

$grad \cdot div A = 7i + 2j + 3k$

v) curl curl A

$$curl\ A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy+yz) \end{vmatrix}$$

$$i\left[\partial/\partial y (xz^2) - \partial/\partial z (xy + yz)\right] - j\left[\partial/\partial x (xz^2) - \partial/\partial z (x^2 y)\right]$$

$$+ k\left[\partial/\partial x (xy + yz) - \partial/\partial y (x^2 y)\right]$$

$$i[0 - y] - j[z^2 - 0] + k(y - x^2)$$

$$= -yi - z^2 j + (y - x^2) k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$i\left[\partial/\partial y(y-x^2) - \partial/\partial z(-z^2)\right] - j\left[\partial/\partial x(y-x^2) - \partial/\partial z(-y)\right]$$

$$+k\left[\partial/\partial x(-z^2) - \partial/\partial y(-y)\right]$$

$$\text{Curl curl } A = i\left[1+2z\right] - j\left[-2x+0\right] + k\left[0+1\right]$$

$$\text{Curl curl } A = (1+2z)i + 2xj + k$$

$$\text{Curl curl } A = (1+2(1))i + 2(1)j + k$$

at point $(1,2,1)$

$$\text{Curl curl } A = 3i + 2j + k$$