1)

i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpreting the result in physical or in order terms.

ii) a) Using Balance Law - Law of conservation of mass.

b) Forming a differential equation from an existing algebraic equation of the system.

2)

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0) j + 12e^{2(0)}k$

$= 2i + 12k$

$$\left.\frac{d^2 r}{dt^2}\right|_{t=0} = 12.17$$

3) at point $(1, 2, 1)$ [x y z]

$$A = x^2 y\, i + (xy + yz)\, j + xz^2 k$$

$$B = yz\, i - 3xz\, j + 2xy\, k$$

$$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\nabla\phi = (6xy + yz)\, i + (3x^2 + xz - 8yz^2)\, j + (xy - 8y^2 z)\, k$$

at point $(1, 2, 1)$

$$\nabla\phi = (6(1)(2) + (2)(1))\, i + ((3(1)^2 + (1)(1) - 8(2)(1)^2)\, j + [(1)(2) - 8(2)^2(1)]\, k$$

$$\nabla\phi = 14i - 12j - 30k$$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$
at $(1,2,1)$

$\nabla \cdot A = 4 + 2 + 2$
$\nabla \cdot A = 8$

(ii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$

$i\left[\partial/\partial y(2xy) - \partial/\partial z(-3xz)\right] - j\left[\partial/\partial x(2xy) - \partial/\partial z(yz)\right]$

$+ k\left[\partial/\partial x(-3xz) - \partial/\partial y(yz)\right]$

$\nabla \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

$\partial iv A = 2xy + (x+z) + 2xz$

$\text{grad}\cdot \partial iv A = \partial/\partial x \frac{\partial A}{\partial x} i + \partial/\partial y\, \partial A/\partial y\, j + \partial/\partial z\, \partial A/\partial z\, k$

$\text{grad}\, d\cancel{iv}A = (2y + 1 + 2z)i + (2x)j + (\cancel{1} + 2x)k$

grad. div A at point (1,2,1) $= (2(2)+1(2(1)))i + (2(1))j + (1+2(1))k$

$\text{grad}\cdot \partial iv A = 7i + 2j + 3k$

v) Curl curl A

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+y^2) & xz^2 \end{vmatrix}$$

$$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+y2) & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+y2) \end{vmatrix}$$

$$i\left[\partial/\partial y(xz^2) - \partial/\partial z(xy+y2)\right] - j\left[\partial/\partial x(xz^2) - \partial/\partial z(x^2y)\right]$$

$$+k\left[\partial/\partial x(xy+y2) - \partial/\partial y(x^2y)\right]$$

$$i[0-y] - j[z^2 - 0] + k(y - x^2)$$

$$= -yi - z^2j + (y-x^2)k$$

$+ k\left[\partial/\partial x(-z^2) - \partial/\partial y(-y)\right]$

$curl\ curl\ A = i[1 + 2z] - j[-2x + 0] + k[0 + 1]$

$curl\ curl\ A = (1 + 2z)i + 2xj + k$

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