Mathematics.

1) Mathematical modelling is the process of using various Mathematical structures (i.e graphs, equations, diagrams, Scatter plots, tree diagrams, e.t.c) to represent real world situations. The model provides an abstraction that reduces a problem to it's essential characteristics

b): Using the balanced law

i: By forming differential equations from an existing algebraic equation of the system.

2) $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t} k$

$dr/dt = (2t+3)i - 6\cos 3t j + 6e^{2t} k$

$d^2r/dt^2 = 2i + 18\sin 3t j + 12e^{2t} k$

at $t = 0$

$d^2r/dt^2 = 2i + 12k$

$|d^2r/dt^2|$ where $t = 0$

$\therefore \sqrt{2^2+12^2} = \sqrt{4+144} = \sqrt{148} = 12.17$

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3) $A = x^2 y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\varnothing = 3x^2 y + xyz - 4y^2 z^2 - 3$

Solution

$\nabla \varnothing = (6xy + yz) i - (3x^2 + xz - 8yz^2) j + (xy - 8y^2 z)$

$\nabla \cdot A = (x^2 y) i \, d i / dx + (xy + yz) j \, d/dy \, J$

$+ (xz^2) k \, d/dz \, k$

$i \cdot i = j \cdot j = k \cdot k = 1$

$= 2xy + x + z + 2xz$

$$\nabla A B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i\left(\frac{d}{dy}(2xy)\right) - \frac{d}{dz}(-3xz)\Big) - J\left(\frac{d}{dy}(2xy)\right.$

$\left.-\frac{d}{dz}(yz)\right) + k\left(\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz^2)\right)$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

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$\nabla \cdot A = x^2 y i \cdot \frac{d}{dx} i + (xy + yz) j \cdot \frac{d}{dy} j + xz^2 k \cdot \frac{d}{dy} k$

$= (i \cdot i = j \cdot j = k \cdot k)$

$= 2xy + x + z + 2xz$

$\nabla(\nabla \cdot A) = (2xy + x + z + 2xz)\frac{d}{dx} i + (2xy + x + z$

$+ 2xz)\frac{d}{dy} j$

$+ (2xy + z + x + 2xz)\frac{d}{dz} k$

$= (2y + 1 + 2z) i + (2x) j + (2x + 1) k$

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$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left(\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right) - j\left(\frac{d}{dx}(xz^2) - \frac{d}{dz}(x\right.$$

$$+ k\left(\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2 y)\right)$$

$$= -z^2 j + (y - x^2) k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left(\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right) - j\left(\frac{d}{dx}(y-x^2 - \frac{d}{dz}(-y)\right)$$

$$+ k\left(\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right)$$