DEPT ELECT/ELECT

(1) mathematical modelling : This is the process of representing a real system, using mathematical equations and structures. The structures may either be in the form of graphs, tree diagrams, Scatterplots, e.t.c. The model provides an abstraction that reduces a problem to its essential characteristics.

(2) Methods of obtaining models for engineering system

(a) Using Balance law.

(b) Forming Differential Equations from an existing algebraic equation of the system.

2

If $r = (t^2 + 3t)i - 2\sin 3t\,j + 3e^{2t}k$

$$\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\,j + 6e^{2t}k$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$$

when $t = 0$; $2i + 12k$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2}$$

$$= \sqrt{4 + 144}$$

$$= \sqrt{148} = \underline{12.17}$$

(3) $A = x^2yi + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

and

$\phi = 3x^2 + xyz - 4y^2z^2 - 3$

Determine at point $(x, y, z) \Rightarrow (1, 2, 1)$

(i) $\nabla\phi$

when $\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$$\nabla\phi = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)\left(3x^2y + xyz - 4y^2z^2 - 3\right)$$

$$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8zy^2)k$$

(ii) $\nabla \cdot A$

$$\left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right) \cdot \left(x^2yi + (xy + yz)j + xz^2k\right)$$

$$\nabla \cdot A = x^2y\frac{d}{dx} + (xy + yz)\frac{d}{dy} + xz^2\frac{d}{dz}$$

$$\nabla \cdot A = 2xy + x + z + 2zx$$

(iii) $\nabla \times B$

$B = yzi - 3xzj + 2xyk$

$$\nabla = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)$$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

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$$\nabla \times B = i\left(\frac{d}{dy} 2xy + \frac{d}{dz} 3xz\right) - j\left(\frac{d}{dx} 2xy - \frac{d}{dz} yz\right)$$

$$+ k\left(\frac{d}{dx}(-3xz) - \frac{d}{dy} yz\right)$$

$$\nabla \times B = (2x + 3x)i - j(2y - y) + k(-3z - z)$$

$$\nabla \times B = 5xi - yj - 4zk$$

(iv) grad div A

$$\nabla \cdot A$$

$$= 2xy + x + z + 2zx$$

$$\text{grad div } A = \nabla(\nabla \cdot A)$$

$$= (2xy + x + z + 2xz)\left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)$$

$$(2xy + x + z + 2xz)i\frac{d}{dx} + (2xy + x + z + 2xz)j\frac{d}{dy} +$$

$$(2xy + x + z + 2xz)k\frac{d}{dz}$$

$$= (2y + 1 + 2z)i + 2xj + (1 + 2x)k$$

Curl curl A

$A = x^2 y\, i + (xy + yz) j + xz^2 k$

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left(\frac{d}{dy} xz^2 - \frac{d}{dz}(xy+yz)\right) - j\left(\frac{d}{dx} xz^2 - \frac{d}{dz} x^2y\right) +$$

$$k\left(\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right)$$

$$= i(0-y) - j(z^2-0) + k(y-x^2)$$

$$-yi \quad - z^2 j \quad + (y-x^2)k$$

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & z^2 & (y-x^2) \end{vmatrix}$$

$$i\left(\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right) - j\left(\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right)$$

$$+ k\left(\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right)$$

$$i(1+2z) - j(-2x) + k(-(-1))$$

$$= i(1+2z) + 2xj + k$$

$$= (1+2z)i + 2xj + k.$$