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DEPT: ELECT/ELECT

(1) Mathematical modelling: This is the process of representing a real system, using mathematical equations and structures. The structures may either be in the form of graphs, tree diagrams, scatterplots, e.t.c. The model provides an abstraction that reduces a problem to its essential characteristics.

(2) Methods of obtaining models for engineering system

(a) Using Balance Law.

(b) Forming Differential Equations from an existing algebraic equation of the system.

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$$\text{If } r = (t^2 + 3t)\mathbf{i} - 2\sin 3t\mathbf{j} + 3e^{2t}\mathbf{k}$$

$$\frac{dr}{dt} = (2t + 3)\mathbf{i} - 6\cos 3t\mathbf{j} + 6e^{2t}\mathbf{k}$$

$$\frac{d^2r}{dt^2} = 2\mathbf{i} + 18\sin 3t\mathbf{j} + 12e^{2t}\mathbf{k}$$

$$\text{When } t = 0; 2\mathbf{i} + 12\mathbf{k}$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{2^2 + 12^2}$$

$$= \sqrt{4 + 144}$$

$$= \sqrt{148} = \underline{12.17}$$

$$(3) A = x^2y\mathbf{i} + (xy + yz)\mathbf{j} + xz^2\mathbf{k}$$

$$B = yz\mathbf{i} - 3xz\mathbf{j} + 2xy\mathbf{k}$$

and

$$\phi = 3x^2 + xyz - 4y^2z^2 - 3$$

Determine at point $(x, y, z) \Rightarrow (1, 2, 1)$

$$(i) \nabla\phi$$

$$\text{when } \phi = 3x^2y + xyz - 4y^2z^2 - 3$$

$$\nabla\phi = \left(\mathbf{i} \frac{d}{dx} + \mathbf{j} \frac{d}{dy} + \mathbf{k} \frac{d}{dz} \right) (3x^2y + xyz - 4y^2z^2 - 3)$$

$$\nabla\phi = (6xy + yz)\mathbf{i} + (3x^2 + xz - 8yz^2)\mathbf{j} + (xy - 8zy^2)\mathbf{k}$$

$$(ii) \nabla \cdot A$$

$$\left(\mathbf{i} \frac{d}{dx} + \mathbf{j} \frac{d}{dy} + \mathbf{k} \frac{d}{dz} \right) \cdot (x^2y\mathbf{i} + (xy + yz)\mathbf{j} + xz^2\mathbf{k})$$

$$\nabla \cdot A = x^2y \frac{d}{dx} + (xy + yz) \frac{d}{dy} + xz^2 \frac{d}{dz}$$

$$\nabla \cdot A = 2xy + x + z + 2zx$$

$$(iii) \nabla \times B$$

$$B = yz\mathbf{i} - 3xz\mathbf{j} + 2xy\mathbf{k}$$

$$\nabla = \left(\mathbf{i} \frac{d}{dx} + \mathbf{j} \frac{d}{dy} + \mathbf{k} \frac{d}{dz} \right)$$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz - 3xz & 2xy & \end{vmatrix}$$

$$\begin{aligned} \nabla \times B &= i \left(\frac{d}{dy} 2xy + \frac{d}{dz} 3xz \right) - j \left(\frac{d}{dx} 2xy - \frac{d}{dz} yz \right) \\ &+ k \left(\frac{d}{dx} (-3xz) - \frac{d}{dy} yz \right) \end{aligned}$$

$$\nabla \times B = (2x + 3x)i - j(2y - y) + k(-3z - z)$$

$$\nabla \times B = 5xi - yj - 4zk$$

(iv) grad div A

$\nabla \cdot A$

$$= 2xy + x + z + 2xz$$

$$\text{grad div A} = \nabla(\nabla \cdot A)$$

$$= (2xy + x + z + 2xz) \left(i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz} \right)$$

$$(2xy + x + z + 2xz) \frac{d}{dx} i + (2x \frac{d}{dy} y + x + z + 2xz) \frac{d}{dy} j +$$

$$(2xy + x + z + 2xz) k \frac{d}{dz}$$

$$= (2y + 1 + 2z)i + 2xj + (1 + 2x)k$$

Curl curl A

$$\Delta = x^2 y_i + (xy + yz)_j + xz^2 k$$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$= i \left(\frac{d}{dy} xz^2 - \frac{d}{dz} (xy + yz) \right) - j \left(\frac{d}{dx} xz^2 - \frac{d}{dz} x^2 y \right) +$$

$$k \left(\frac{d}{dx} (xy + yz) - \frac{d}{dy} (xz^2) \right)$$

$$= i(0 - y) - j(z^2 - 0) + k(y - x^2)$$
$$= -y i - z^2 j + (y - x^2) k$$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & z^2 & (y - x^2) \end{vmatrix}$$

$$= i \left(\frac{d}{dy} (y - x^2) - \frac{d}{dz} (-z^2) \right) - j \left(\frac{d}{dx} (y - x^2) - \frac{d}{dz} (-y) \right)$$
$$+ k \left(\frac{d}{dx} (-z^2) - \frac{d}{dy} (-y) \right)$$

$$= i(1 + 2z) - j(-2x) + k(-(-1))$$

$$= i(1 + 2z) + 2xj + k$$

$$= (1 + 2z)i + 2xj + k$$