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16/ENG 04 / 018

ELECTRICAL / ELECTRONICS

Question 1

(C) Mathematical modeling can be defined as the process of using mathematical equations and interpreting the result in physical or other terms (simulation).

(P) Methods include -

(a) Forming a differential equation from an existing algebraic equation of the system.

(b) Using law of Conservation of mass.

Question 2

$$\vec{r} = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$$

$$(i) \frac{d\vec{r}}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$$

$$(ii) \frac{d^2\vec{r}}{dt^2} = \frac{d}{dt} \left(\frac{d\vec{r}}{dt} \right) = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$$

$$(iii) \left| \frac{d^2\vec{r}}{dt^2} \right| \text{ at } t=0$$

$$\left| \frac{d^2\vec{r}}{dt^2} \right| \text{ at } t=0 = 2\hat{i} + 18\sin(3 \cdot 0)\hat{j} + 12e^{2 \cdot 0}\hat{k}$$
$$\frac{d^2\vec{r}}{dt^2} \text{ at } t=0 = 2\hat{i} + 12\hat{k}$$

$$\left| \frac{d^2 r}{dt^2} \right| = \sqrt{(2)^2 + (2)^2}$$

$$t=0 \quad \left| \frac{d^2 r}{dt^2} \right|_{t=0} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Question 3

$$A = x^2 y \hat{i} + (x y + y z) \hat{j} + x z^2 \hat{k}$$

$$B = y z \hat{i} - 3 x z \hat{j} + 2 x y \hat{k} \quad \text{one}$$

$$\phi = 3 x^2 y + x y z - 4 y^2 z^2 - 3$$

$$(1, 2, 1)$$

$$\nabla \phi$$

$$\nabla \phi = \frac{d\phi}{dx} \hat{i} + \frac{d\phi}{dy} \hat{j} + \frac{d\phi}{dz} \hat{k}$$

$$\nabla \phi = (6xy + yz) \hat{i} + (3x^2 + xz - 8yz^2) \hat{j} + (xy - 8y^2 z) \hat{k}$$

at point (1, 2, 1)

$$\nabla \phi = (6(1)(2) + (2)(1)) \hat{i} + (3(1)^2 + (1)(2) - 8(2)(1)^2) \hat{j} + ((1)(2) - 8(2)^2(1)) \hat{k}$$

$$\nabla \phi = 14 \hat{i} - 12 \hat{j} - 30 \hat{k}$$

~~$$\nabla \phi = 14 \hat{i} - 12 \hat{j} - 30 \hat{k}$$~~

$$\nabla \phi = 2(7 \hat{i} - 6 \hat{j} - 15 \hat{k})$$

$$\nabla \cdot A =$$

$$\frac{d}{dx} + \frac{d}{dy} + \frac{d}{dz}$$

$$\nabla \cdot A = 2xy + (x+z) + 2xz$$

$$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$$

$$\nabla \cdot A = 8$$

(11)

$\nabla \times B$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\nabla \times B = \hat{i} \left[\frac{d}{dy} (2xy) - \frac{d}{dz} (-3xz) \right] - \hat{j} \left[\frac{d}{dx} (2xy) - \frac{d}{dz} (yz) \right] + \hat{k} \left[\frac{d}{dx} (-3xz) - \frac{d}{dy} (yz) \right]$$

$$\nabla \times B = \left(\frac{d}{dy} (2xy) - \frac{d}{dz} (-3xz) \right) \hat{i} - \left(\frac{d}{dx} (2xy) - \frac{d}{dz} (yz) \right) \hat{j} + \left[\frac{d}{dx} (-3xz) - \frac{d}{dy} (yz) \right] \hat{k}$$

$$\nabla \times B = \hat{i} (2x + 3x) - \hat{j} (2y - y) + \hat{k} (-3z - z)$$

$$\nabla \times B = 5x\hat{i} - y\hat{j} - 4z\hat{k}$$

at point (1, 2, 1)

$$\nabla \times B = 5(1)\hat{i} - 2\hat{j} - 4(1)\hat{k}$$

$$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$$

(12)

$$\text{grad div } A = \nabla \cdot (\nabla A) - \nabla^2 A$$

$$\text{grad div } A = \frac{d}{dx} \frac{dA}{dx} \hat{i} + \frac{d}{dy} \frac{dA}{dy} \hat{j} + \frac{d}{dz} \frac{dA}{dz} \hat{k}$$

$$A = x^2y \hat{i} + (xy + yz) \hat{j} + 2xz \hat{k}$$

$$\text{div } A = 2xy + (x + z) + 2xz$$

$$\text{grad div } A = \frac{d}{dx} \frac{dA}{dx} \hat{i} + \frac{d}{dy} \frac{dA}{dy} \hat{j} + \frac{d}{dz} \frac{dA}{dz} \hat{k}$$

$$\text{grad div } A = (2y + 1 + 2z) \hat{i} + (2x) \hat{j} + (1 + 2x) \hat{k}$$

$$\text{grad div } A = (2(2) + 1 + 2(1)) \hat{i} + 2(1) \hat{j} + (1 + 2(2)) \hat{k}$$

$$\text{grad div A} = 7\hat{i} + 2\hat{j} + 3\hat{k}$$

(L) Curl Curl A

$$\text{Curl A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix} = \hat{i} \begin{vmatrix} \frac{d}{dz} & \frac{d}{dz} \\ x^2y & xz^2 \end{vmatrix} - \hat{j} \begin{vmatrix} \frac{d}{dx} & \frac{d}{dz} \\ x^2y & xz^2 \end{vmatrix} + \hat{k} \begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ x^2y & (xy+yz) \end{vmatrix}$$

$$\text{Curl A} = \hat{i} \left[\frac{d}{dz} (xz^2) - \frac{d}{dz} (x^2y) \right] - \hat{j} \left[\frac{d}{dx} (xz^2) - \frac{d}{dz} (x^2y) \right] + \hat{k} \left[\frac{d}{dx} (xy+yz) - \frac{d}{dy} (x^2y) \right]$$

$$\text{Curl A} = y\hat{i} - 2z\hat{j} + (y-x^2)\hat{k}$$

$$\text{Curl Curl A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -2z & (y-x^2) \end{vmatrix}$$

$$\text{Curl Curl A} = \hat{i} \left[\frac{d}{dz} (y-x^2) - \frac{d}{dz} (-2z) \right] - \hat{j} \left[\frac{d}{dx} (y-x^2) - \frac{d}{dz} (-y) \right] + \hat{k} \left[\frac{d}{dx} (-2z) - \frac{d}{dy} (-y) \right]$$

$$\text{Curl Curl A} = \hat{i} (1+2z) - \hat{j} (-2x+0) + \hat{k} [0+1]$$

$$\text{Curl Curl A} = (1+2z)\hat{i} + 2x\hat{j} + \hat{k}$$

$$\text{Curl Curl A} (1,2,1) = (1+2(1))\hat{i} + 2(1)\hat{j} + \hat{k}$$

$$\text{Curl Curl A} = 3\hat{i} + 2\hat{j} + \hat{k}$$