Olatunbosun 'Basit 'O

16/ENG04/043

Electrical electronics.

ENG 282.

05/04/2018.

1.) i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpreting the result in physical or in order terms.

ii) a. Using Balance law - Law of conservation of mass

b. forming a differential equation from an existing algebraic equation of the system.

2.) $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k.$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k.$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) =$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k.$

$$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k.$$

$= 2i + 18\sin 0 j + 12e^0 k$

$= 2i + 18\times 0 j + 12\times 1 k$

$= 2i + 12k$

iv) $\left|\frac{d^2r}{dt^2}\right| = |2i + 12k|$

$= \sqrt{(2i)^2 + (12k)^2}$

$= \sqrt{4\times i\cdot i + 144\times k\cdot k} = \sqrt{4\times 1 + 144\times 1}$

$= \sqrt{4+144} = \sqrt{148}.$

v) $\left|\frac{d^2r}{dt^2}\right|_{t=0} = 2\sqrt{37}$

$\left|\frac{d^2r}{dt^2}\right| = 2\sqrt{37} = 12.16552506$

$\simeq 12.17$

3.) at point $(1, 2, 1)$

$A = x^2y\, i + (xy + yz)\, j + xz^2 K.$

$B = yz i - 3xz j + 2xy K.$

$\phi = 3x^2y + xyz - 4y^2 z^2 - 3.$

i.) $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial}{\partial z}\phi k$

$\nabla\phi = \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial x} i + \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial y} j + \frac{\partial(3x^2y}{}$

$+ \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial z} k$

$\nabla\phi = (6xy + yz) i + (3x^2 + xz - 8yz^2) j + (xy - 8y^2z) K.$

at point $\overset{x}{1}, \overset{y}{2}, \overset{z}{1}$

$\nabla\phi = (6(1)(2) + (2)(1))\, i + (3(1^2) + (1)(1) - 8(2)(1^2)) j + ((1)(2) - 8(2^2)(1)) k$

$\nabla\phi = (12 + 2) i + (3 + 1 - 16) j + (2 - 32) k$

$\nabla\phi_{(1,2,1)} = 14 i - 12 j - 30 k$

ii.) $\nabla\cdot A = \frac{\partial}{\partial x}\cdot A i + \frac{\partial}{\partial y}\cdot A j + \frac{\partial}{\partial z}\cdot A k.$

$\nabla\cdot A = \frac{\partial}{\partial x}(x^2y) i\cdot i +)_0$

$\nabla A = 2xy\, i\cdot i + (x + z) j\cdot j + 2xz\, k\cdot k$

$= 2xy + (x + z) + 2xz$

$\nabla A(1, 2, 1) = 2(1)(2) + (1)+(1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

$\nabla A_{1,2,1} = \underline{8}$

$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= \left(\frac{\partial}{\partial y}\cdot 2xy - \frac{\partial}{\partial z}\cdot -3xz\right) i - j\left(\frac{\partial}{\partial x}\cdot 2xy - \frac{\partial}{\partial z}\cdot yz\right) + k\left(\frac{d}{\partial x}\cdot -3xz - \frac{\partial}{\partial y}\cdot yz\right)$

Scanned by CamScanner

$$\nabla \times B = \left(2x - (-3x)\right)i \quad - j\left(2y - y\right) \quad + k\left(\overset{-3z}{\cancel{-6x}} - z\right)$$

$$\nabla \times B = 5x\, i \quad - yj \quad - k\,(\overset{4z}{\cancel{6x + 4}})$$

$\nabla \times B = 5xi - yj - 4zk.$

1,2,1 $= \cancel{5(1)i - y}$

$5(1)i - (2)j - 4(1)k$

$= \underline{5i - 2j - 4k}$

(iv). $\underline{\text{grad.div} A} = \nabla \cdot \nabla A$

$$= \frac{\partial}{\partial x} \cdot \nabla A\, i + \frac{\partial}{\partial y} \cdot \nabla A j + \frac{\partial}{\partial z} \cdot \nabla A\, k.$$

recall $\nabla A = 2xy + x + z + 2xz$

~~grad~~ $\nabla \cdot \nabla A = \frac{\partial}{\partial x}(2xy + x + z + 2xz) \cdot i + \frac{\partial}{\partial y}(2xy + x + z + 2xz)j + \frac{\partial}{\partial z}(2xy + x + z + 2xz)$

$\nabla \cdot \nabla A = (2y + 1 + 2z) \cdot i + (2x)j + (1 + 2x)k$

at point (1,2,1)

$= 2(2) + 1 + 2(1)i + 2(1)j + 1 + 2(1)k$

$(4 + 1 + 2)i + 2j + (1 + 2)k$

$\nabla \cdot \nabla A = 7i + 2j + 3k.$

grad. div $A = \underline{7i + 2j + 3k}.$

curl curl A = curl (curl A)

Curl A = $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$

curl A = $i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial z & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+yz) \end{vmatrix}$

curl A = $i\left(\frac{\partial}{\partial y} \cdot xz^2 - \frac{\partial}{\partial z}(xy+yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right) + k\left(\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right)$

$= i(0-y) + -j(z^2-0) + k(y-x^2)$

curl A $= -yi - z^2j + (y-x^2)k.$

curl A $= \underline{-yi - z^2j + (y-x^2)k}$

curl curl A = $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y-x^2 \end{vmatrix}$

$i\left(\frac{\partial}{\partial y} \cdot (y-x^2) - \frac{\partial}{\partial z} \cdot -z^2\right) - j\left(\frac{\partial}{\partial x} \cdot y-x^2 - \frac{\partial}{\partial z} \cdot y\right) + k\left(\frac{\partial}{\partial x} \cdot z^2 - \frac{\partial}{\partial y} \cdot (-y)\right)$

curl curl A = $\cancel{1-2x^2} - \cancel{2z}$

$= (1+2z)i - j(-2x-0) + k(0-(-1))$

curl curl A $= (1+2z)i + 2xj + k.$

curl curl A $= (1+2z)i + 2xj + k$

$= (1+2(1))i + 2(1)j + k$

$= 3j + 2j + k = 3i + 2j + k$

curl curl A $= \underline{3i + 2j + k}$

Scanned by CamScanner