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16/ENG01/017
Chemical Engineering
ENG 282
Assignment III

(1) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

b i) Exponential growth/decay (Use of ODE)

b ii) Mixing Problems

(2) $r = (t^2 + 3t)\hat{i} - 2\sin 3t\,j + 3e^{2t}k$

i) $\dfrac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\,\hat{j} + 6e^{2t}\hat{k}$

ii) $\dfrac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\,\hat{j} + 12e^{2t}\hat{k}$

iii) $\left.\dfrac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$= \sqrt{2^2 + 12^2} = \sqrt{148} = 2\sqrt{37} = 12.17$

(3) $A = x^2y\,i + (xy + yz)j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \dfrac{\partial\phi}{\partial x}i + \dfrac{\partial\phi}{\partial y}j + \dfrac{\partial\phi}{\partial z}\hat{k}$

$\dfrac{d\phi}{dx} = 6xy + yz \qquad \dfrac{d\phi}{\partial y} = 3x^2 - xz - 8yz^2$

$\dfrac{\partial\phi}{\partial z} = xy - 8y^2z$

At $(\overset{x}{1}, \overset{y}{2}, \overset{z}{1})$

$$\frac{\delta\phi}{\delta x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\delta\phi}{\delta y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3+1-16 = -12$$

$$\frac{\delta\phi}{\delta z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}.$$

ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

At $(1,2,1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4+2+2 = 8.$

iii) $\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

At $(1,2,1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}.$

iv) Gradient A

grad $(2xy + (x+z) + 2xz)$

Let div $A = C = \nabla A.$

$\nabla(\nabla A) = \nabla C = \hat{i}\frac{\partial c}{\partial x} + \hat{j}\frac{\partial c}{\partial y} + \hat{k}\frac{\partial c}{\partial z}.$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2\cancel{k}) + \hat{k}(1 + 2k)$

At $(1,2,1)$

$$\nabla C = i(2(2)+2(1)) + \hat{j}(2(1)) + \hat{k}(1+(2)(1))$$
$$= i(4+1+2) + \hat{j}(2) + \hat{k}(1+2)$$
$$= 7\hat{i} + 2\hat{j} + 3\hat{k}$$

v) Curl(curl A)

curl A = $\nabla \times A$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy+y^2) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0-y) - \hat{j}(z^2-0) + \hat{k}(y-x^2)$$
$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y-x^2)$$

At (1,2,1).

curl A $= -2\hat{i} - \hat{j} + \hat{k}$

curl(curl A) = $\nabla \times (\nabla \times A)$.

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}(1+2z) - \hat{j}(-2x^2-0) + \hat{k}(0+1)$$
$$= \hat{i}(1+2z) + 2x^2\hat{j} + \hat{k}$$

At point (1,2,1)

$$\nabla \times (\nabla \times A) = \hat{i}(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$$
$$= 3\hat{i} + 2\hat{j} + \hat{k}.$$

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