

Akintan Oluwamodupe Esther

16/MH501/029

Computer Engineering

Assignment 3

Question 1

(a) Modelling can be defined as the process of setting up a model solving it mathematically, and interpreting the result in physical or other terms

(b) Methods of creating models

(i) Radioactivity exponential growth/decay

(ii) Mixing problems

Question 2

$$r = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$$

(i)  $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

(ii)  $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

(iii)  $\frac{d^2r}{dt^2}$  at  $t=0$   
 $= 2\hat{i} + 18\sin 0\hat{j} + 12e^{2 \times 0}\hat{k}$

$$\frac{d^2r}{dt^2} = 2\hat{i} + 12\hat{k}$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{2^2 + 12^2}$$

$$\left| \frac{d^2r}{dt^2} \right| = 12.17$$

Question 3

$$A = xy\hat{i} + (xy + yz)\hat{j} + xz\hat{k}$$

$$B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$$

$$\phi = 3x^2y + xyz - 4y^2z^2 - 3$$

at point (1, 2, 1)

(i)  $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (2xy - 8y^2z)\hat{k}$$

at point (1, 2, 1)

$$\nabla\phi = (12 + 2)\hat{i} + (3 + 1 - 16)\hat{j} + (2 - 32)\hat{k}$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$$

$$(10) \nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$\nabla \cdot A = 2xy + (x+2) + 2xz$$

at point (1, 2, 1)

$$\nabla \cdot A = 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

$$(11) \nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ b_x & b_y & b_z \end{vmatrix}$$

$$\nabla \times B = \hat{i} \left( \frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z} \right) - \hat{j} \left( \frac{\partial b_z}{\partial x} - \frac{\partial b_x}{\partial z} \right) + \hat{k} \left( \frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y} \right)$$

$$\frac{\partial b_x}{\partial y} = 2x, \quad \frac{\partial b_y}{\partial z} = -3x, \quad \frac{\partial b_z}{\partial x} = 2y$$

$$\frac{\partial b_z}{\partial z} = y, \quad \frac{\partial b_y}{\partial x} = -3z, \quad \frac{\partial b_x}{\partial y} = z$$

$$\nabla \times B = (2x - (-3x))\hat{i} - (2y - y)\hat{j} + (-3z - z)\hat{k}$$

$$\nabla \times B = 5x\hat{i} - y\hat{j} - 4z\hat{k}$$

at point (1, 2, 1)

$$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$$

$$(12) \text{grad div } A = \frac{\partial (\nabla \cdot A)}{\partial x} \hat{i} + \frac{\partial (\nabla \cdot A)}{\partial y} \hat{j} + \frac{\partial (\nabla \cdot A)}{\partial z} \hat{k}$$

where  $\nabla \cdot A = 2xy + (x+2) + 2xz$

$$\nabla(\nabla \cdot A) = (2y + 1 + 2z)\hat{i} + (2x)\hat{j} + (1 + 2x)\hat{k}$$

at point (1, 2, 1)

$$\nabla(\nabla \cdot A) = 7\hat{i} + 2\hat{j} + 3\hat{k}$$

(v)  $\text{Curl curl } A = \nabla \times (\nabla \times A)$  let  $\nabla \times A = C$

$$\nabla \times C = \left( \frac{\partial C_z}{\partial y} - \frac{\partial C_y}{\partial z} \right) \hat{i} - \left( \frac{\partial C_z}{\partial x} - \frac{\partial C_x}{\partial z} \right) \hat{j} + \left( \frac{\partial C_y}{\partial x} - \frac{\partial C_{xz}}{\partial y} \right) \hat{k}$$

$$\nabla \times A = \left( \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z} \right) \hat{i} - \left( \frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z} \right) \hat{j} + \left( \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y} \right)$$

$$A = x^2 y \hat{i} + (xy + yz) \hat{j} + xz^2 \hat{k}$$

$$\frac{\partial a_z}{\partial y} = 0, \quad \frac{\partial a_y}{\partial z} = y, \quad \frac{\partial a_x}{\partial x} = z^2$$

$$\frac{\partial a_x}{\partial z} = 0, \quad \frac{\partial a_y}{\partial x} = y, \quad \frac{\partial a_x}{\partial y} = x^2$$

$$\therefore \nabla \times A = (0 - y) \hat{i} - (z^2 - 0) \hat{j} + (y - x^2) \hat{k}$$

$$\nabla \times A = -y \hat{i} - z^2 \hat{j} + (y - x^2) \hat{k}$$

where  $\nabla \times A = C$

$$\frac{\partial C_z}{\partial y} = 1, \quad \frac{\partial C_y}{\partial z} = -2z, \quad \frac{\partial C_x}{\partial x} = -2x$$

$$\frac{\partial C_x}{\partial z} = 0, \quad \frac{\partial C_y}{\partial x} = 0, \quad \frac{\partial C_x}{\partial y} = -1$$

$$\therefore \nabla \times C = (1 - (-2x)) \hat{i} - (-2z - 0) \hat{j} + (0 - (-1)) \hat{k}$$

$$\nabla \times C = (1 + 2x) \hat{i} + 2z \hat{j} + \hat{k}$$

$\therefore$  at point  $(1, 2, 1)$

$$\nabla \times C = \nabla \times (\nabla \times A) = (1 + 2) \hat{i} + 2 \hat{j} + \hat{k}$$

$$\therefore \nabla \times (\nabla \times A) = 3 \hat{i} + 2 \hat{j} + \hat{k}$$