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Course: ENG 282 (Assignment III)

Q1

a) Define mathematical modelling

= Mathematical modelling is a general process in Engineering, Physics, Computer Science, biology, medicine, environmental science, Chemistry, economics and other fields that translates a physical situation or some other observations into Mathematical equations

b) Outline two methods of obtaining models for engineering Systems.

(i) - Using the Balance law

(ii) - forming a differential equation from an existing algebraic equation of the System

Q2

If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$.

Determine

(i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t} k$

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k$.

(iii) $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$ $= 2i + 18\sin(3 \times 0)j + 12e^{2(0)} k$

$= 2i + 12k$

$= \sqrt{2^2 + 12^2}$

$= 2\sqrt{37} = 12.17$

Q's

i) If

$A = x^2 y i + (xy + yz) j + x z^2 k$

$B = yz i - 3xz j + 2xy k$ and

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

Determine, at Point (1, 2, 1)

(i) $\nabla \phi =$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

$= (6xy + yz) i + (3x^2 + xz - 8yz^2) j + (xy - 8y^2 z) k$

$= (6(1)(2) + (2)(1)) i + (3(1)^2 + (1)(1) - 8(2)(1)^2) j + ((1)(2) - 8(2)^2(1)) k$

$= 14i + (-12) j - 30k$

$= 14i - 12j - 30k$

(ii) $\nabla \cdot A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right) \cdot \left(x^2 y i + (xy + yz) j + x z^2 k \right)$

$= \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(x z^2)$

$= 2xy + (x + z) + 2xz$

$= x(2y + 1) + z(2x + 1) = \underline{8}$

(iii) $\nabla \times B = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right) \times \left(B_x i + B_y j + B_z k \right)$

$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right) \times \left(yz i - 3xz j + 2xy k \right)$

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$$\nabla \times B = \begin{vmatrix} i & -j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= i\left[\frac{\partial 2xy}{\partial y} + \frac{\partial 3xz}{\partial z}\right] - j\left[\frac{\partial 2xy}{\partial x} - \frac{\partial yz}{\partial z}\right] + \left[\frac{-\partial -3xz}{\partial x} - \frac{\partial yz}{\partial y}\right]k$$

$$= i\left[2x + 3x\right] - j\left[2y - y\right] + \left[-3z - z\right]k$$

$$= 5xi - yj + (-4z)k$$

$$= 5i - 2j - 4k$$

14) grad div A

$$\nabla \cdot A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot \left(x^2yi + (xy + yz)j + xz^2k\right)$$

$$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$$

$$\nabla \cdot A = 2xy + (x + z) + 2xz$$

$$\text{grad div } A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left(2xy + (x+z) + 2xz\right)$$

$$= (2y + 2z + 1)i + (2x)j + (2x + 1)k$$

$$= 7i + 2j + 3k$$

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vi) Curl Curl A =

$$\text{Curl} A = \begin{vmatrix} i & -j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & xy+yz & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial xz^2}{\partial y} - xy+yz\frac{\partial}{\partial z}\right] - \left[\frac{\partial xz^2}{\partial x} - x^2y\frac{\partial}{\partial z}\right]j$$

$$+ \left[\frac{\partial xy+yz}{\partial x} - x^2y\frac{\partial}{\partial y}\right]k$$

$$= i[0-0-y] - j[z^2-0] + [y+0-x^2]k$$

$$= -yi - z^2j + (y-x^2]k$$

Curl Curl A =

$$\begin{vmatrix} i & -j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -z^2 & y-x^2 \end{vmatrix}$$

$$= i\left[\frac{\partial y-x^2}{\partial y} + z^2\frac{\partial}{\partial z}\right] - \left[\frac{\partial y-x^2}{\partial x} - y\frac{\partial}{\partial z}\right]j$$

$$+ \left[\frac{\partial z^2}{\partial x} - y\frac{\partial}{\partial y}\right]$$

$$= i[-x^2+2z] - (0-2x-0]j + [0+1]k$$

$$= [1+2]i + [2]j + k$$

$$= 3i + 2j + k$$