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Course: ENG 282.

No. 1

1(i) Mathematical Modelling can be defined as the process of setting up a model of an engineering problem, solving it mathematically and interpreting the result in ~~terms of~~ physical or other terms.

This model is the formulation of the problem as a mathematical expression in terms of variables, functions and equations.

(ii) Two methods of obtaining models for engineering systems:

a. ~~MALTHUS~~ Malthu's law.

b. Experimental decay.

No. 2

$$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$$

(i) $$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$$

(ii) $$\frac{d^2r}{dt^2} = 2i + 18\sin \cancel{\cos} 3t\, j + 12e^{2t}k$$

(iii) at $t=0$, $$\frac{d^2r}{dt^2} = 2i + 18\sin(3\times 0)j + 12e^{2(0)}k$$

$$= 2i + 12k$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2 + (12)^2}$$

$$= \sqrt{4+144}$$

$$= \sqrt{148}$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$$

No 3

$A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

at the point $(1,2,1)$

(i) $\nabla\phi = \text{grad}\,\phi = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot(3x^2y + xyz - 4y^2z^2 - 3)$

$$= \frac{\partial}{\partial x}(3x^2y + xyz - 4y^2z^2 - 3)\,i + \frac{\partial}{\partial y}(3x^2y + xyz - 4y^2z^2 - 3)\,j$$

$$+ \frac{\partial}{\partial z}(3x^2y + xyz - 4y^2z^2 - 3)\,k$$

$$= i\,(6xy + yz) + j\,(3x^2 + xz - 8yz^2) + k\,(xy - 8y^2z)$$

at the point $(1,2,1)$

$$\nabla\phi = i\,[6(1)(2) + (2)(1)] + j\,[3(1)^2 + (1)(1) - 8(2)(1)^2] + k\,[(1)(2) - 8(2)^2(1)]$$

$$= i\,(12+2) + j\,(3+1-16) + k\,(2-32)$$

$$= 14i - 12j - 30k$$

(ii) $\text{div}\,A = \nabla\cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot\left(x^2y\,i + (xy + yz)\,j + xz^2\,k\right)$

$$\nabla\cdot A = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy+yz) + \frac{\partial}{\partial z}(xz^2)$$

$$= 2xy + (x+z) + 2xz$$

at $(1,2,1)$

$$= 2(1)(2) + (1+1) + 2(1)(1) = 4 + 2 + 2$$

$$\nabla\cdot A = 8$$

(iii) $\text{Curl}\cdot B = \nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$

$$\nabla \times B = i\left[\frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$$

$$= i\,(2x + 3x) - j\,(2y - y) + k(-3z - z)$$

$$= 5x\,i - y\,j - 4z\,k$$

at $(1,2,1)$

$$\nabla \times B = 5(1)i - (2)j - 4(1)k$$

$$= 5i - 2j - 4k$$

(iv) $\text{grad div } A = \nabla(\nabla \cdot A)$

$$\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot x^2y\,i + (xy + yz)\,j + xz^2\,k$$

$$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$$

$$\nabla \cdot A = 2xy + (x + z) + 2xz$$

$$\nabla(\nabla \cdot A) = \left(\frac{\partial}{\partial x}i + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot 2xy + (x+z) + 2xz$$

$$= \frac{\partial}{\partial x}(2xy + (x+z) + 2xz)\,i + \frac{\partial}{\partial y}(2xy + (x+z) + 2xz)$$

$$+ \frac{\partial}{\partial z}(2xy + (x+z) + 2xz)\,k$$

$$= i(2y + 1 + 2z) + \cancel{(2x+1) + k(}\; 2x\,j + (1+2x)k$$

at $(1,2,1)$

$$\nabla(\nabla \cdot A) = [(2)(2) + 1 + 2(1)]\,i + 2(1)\,j + [1 + 2(1)]\,k$$

$$= (4 + 1 + 2)\,i + 2j + (1+2)k$$

$$= 7i + 2j + 3k$$

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(v) curl curl $A = \nabla \times (\nabla \times A)$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz^2) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz^2)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(xy+yz^2) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$\cancel{= i\,(-(x+z))}$$

$$\nabla \times A = i\,(-y) - j\,(z^2) + k\,(y - x^2)$$

$$\cancel{A\ (1,2,1)}$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y-x^2) + \frac{\partial}{\partial z}(z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) + \frac{\partial}{\partial z}(y)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(z^2) + \frac{\partial}{\partial y}(y)\right]$$

$$\nabla \times (\nabla \times A) = i\,(1+2z) - j\,(-2x) + k\,(1)$$

$$= \quad \text{at } (1,2,1)$$

$$= i\,(1+2(1)) - j\,(-2(1)) + k$$

$$= i\,(1+2) - j\,(-2) + k$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k$$