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16/ENG01/002
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Question 1

i) Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

ii) In Chemical engineering → Chemical equilibrium

In Electrical engineering → Power supply network optimization

Question 2

$r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

$\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, j + 6e^{2t} k$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k$

$\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$

$= 2i + 18\sin(3 \times 0)j + 12e^0 k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{148}$

$\left|\frac{d^2r}{dt^2}\right| = 12.17$

Question 3

$A = x^2y\,i + (xy + yz)\,j + xz^2\,\hat{k}$

$B = yz\,i - 3xz\,j + 2xy\,\hat{k}$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi$ at $(1, 2, 1)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\,i + \frac{\partial\phi}{\partial y}\,j + \frac{\partial\phi}{\partial z}\,\hat{k}$$

$$\frac{\partial\phi}{\partial x} = 6xy + yz$$

$$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$$

$$\frac{\partial\phi}{\partial z} = xy - 8y^2z$$

$$\nabla\phi = (6xy + yz)\,\hat{i} + (3x^2 + xz - 8yz^2)\,\hat{j} + (xy - 8y^2z)\,\hat{k}$$

at point $(1, 2, 1)$

$$\nabla\phi = (12 + 2)\,\hat{i} + (3 + 1 - 8(2)(1))\,\hat{j} + (2 - 32)\,\hat{k}$$

$$= 14\hat{i} + (4 - 16)\,\hat{j} + (-30)\,\hat{k}$$

$$= 14i - 12j - 30k$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$$

ii) $\nabla \cdot A = \frac{\partial}{\partial x}\,i + \frac{\partial}{\partial y}\,j + \frac{\partial}{\partial z}\,\hat{k} \cdot (x^2y\,\hat{i} + (xy + y$

$+ xz^2\,\hat{k})$

$$\nabla \cdot A = \frac{\partial (x^2 y)}{\partial x} + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$$

$$\nabla \cdot A = 2xy + (x+z) + 0 + (2xz)$$
$$= 4 + 2 + 2 = 8_{//}$$
$$\nabla \cdot A = 8_{//}$$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial}{\partial y}(2xy) + \frac{\partial(3xz)}{\partial z}\right) - \hat{j}\left(\frac{\partial(2xy)}{\partial x} - \frac{\partial}{\partial z}(yz)\right)$$

$$\hat{k}\left(\frac{\partial}{\partial z}(-3xz) - \frac{\partial}{\partial y}(yz)\right)$$

$$\hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(3z - z)$$
$$(3x)\hat{i} - y(\hat{j}) + (-4z)\hat{k}$$

$$3x\hat{i} - y\hat{j} - 4z\hat{k}$$

at point $(1, 2, 1)$

$$\nabla \times B = 3\hat{i} - 2\hat{j} - 4\hat{k}$$

iv) grad div A. $\nabla \cdot \vec{A}$

$$\text{Div } A = 2xy + (x+z) + 2zi$$

$$\nabla(\nabla \cdot A) = \frac{\partial(\nabla \cdot A)}{\partial x}\hat{i} + \frac{\partial(\nabla \cdot A)}{\partial y}\hat{j} + \frac{\partial(\nabla \cdot A)}{\partial z}\hat{k}$$

$$\nabla(\nabla \cdot A) = \hat{i}(2y + 1 + 2z) + 2x\hat{j} + (1 + 2x)\hat{k}$$
$$= \hat{i}(4 + 1 + 2) + 2\hat{j} + 3\hat{k}$$
$$= 7i + 2\hat{j} + 3\hat{k}$$

$$\nabla \times (\nabla \times \vec{A}) \quad \vec{A} = x^2 y i + (xy + yz)\hat{j} + xz^2\hat{k}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & (xz^2) \end{vmatrix}$$

$$\hat{i}(0 - y) - \hat{j}(z^2 - 0) + k(x^2 + (-y))$$

$$-y\hat{i} - \hat{j}(z^2) + \hat{k}(x^2 - y)$$

$$\nabla \times \vec{A} = -y\hat{i} - \hat{j}(z^2) + \hat{k}(x^2 - y)$$

$$\nabla \times \vec{A}$$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & x^2 - y \end{vmatrix}$$

$$= i(1 + 2z) - j(0 - 2x) + \hat{k}(1 - 0)$$

$$\nabla \times (\nabla \times \vec{A}) = (1 + 2z)\hat{i} + 2x\hat{j} + \hat{k}$$

$$= (1 + 2(1))\hat{i} + 2\hat{j} + \hat{k}$$

$$\nabla \times (\nabla \times \vec{A})$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$