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16/ENG04/039
Electrical/Electronics

Question 1:

(i) Mathematical modelling can be defined as the process of using mathematical equations and interpreting the result in physical or other terms (simulation).

(ii) Method include:
(a) forming a differential equations from an existing algebraic equation of the system
(b) Using law of conservation of mass

Question 2:

$r = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}K$

(i) $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}K.$

(ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = 2i + 18\sin 3t\hat{j} + 12e^{2t}K$

(iii) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2\hat{i} + 18\sin 3(0)\hat{j} + 12e^{2(0)}K$

$\frac{d^2r}{dt^2} = 2\hat{i} + 12K.$

$\left|\frac{d^2r}{dt^2}\right|_{t=0}$ $\left|\frac{d^2r}{dt^2}\right|_{t=0}$ $= \sqrt{(2)^2 + (12)^2}$

$= \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37}$

Question 3

$A = x^2y\hat{i} + (xy + yz)\hat{j} + xz\hat{K}$

$B = yz\hat{i} - 3xz\hat{j} + 2xyK$ and

$\phi = x^2y + xyz - 4y^2z^2 - 3$

$(1, 2, 1)$

(i) $\nabla\phi$

$$\nabla\phi = \frac{d\phi}{dx} i + \frac{\partial\phi}{dy} j + \frac{d\phi}{dz} k$$

$$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2z)k$$

at point $(1,2,1)$

$$\nabla\phi = (6(1)(2) + (2)(1))\hat{i} + (3(1)^2 + (1)(1) - 8(2)(1))\hat{j} + (1)(2) - 8(2)$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30k$$

$$\nabla\phi = 2(7\hat{i} - 6\hat{j} - 15k)$$

(ii) $\nabla \cdot A =$

$$\frac{\partial dx}{dx} + \frac{\partial dy}{dy} + \frac{\partial \partial z}{dz}$$

$$\bar{\nabla} \cdot A = 2xy + (x + z) + 2xz$$

$$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$$

$$\nabla \cdot A = 8$$

(iii) $\nabla \times B$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\nabla \times B = i \begin{vmatrix} \partial/\partial y & \frac{d}{dz} \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial x \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \frac{d}{dx} & \partial/\partial y \\ yz & -3xz \end{vmatrix}$$

$$\nabla \times B = \left[\frac{d}{dy}(2xy) - \frac{d}{z}(-3xz)\right] - j\left[\frac{d}{dx}(2x) - \frac{d}{dz}(yz)\right] + k\left[\frac{d}{dx}(3xz) - \frac{d}{dy}(yz)\right]$$

$$\nabla \times B = i(2x + 3x) - \hat{j}(2y - y) + k(-3z - z)$$

$$\nabla \times B = 5x\hat{i} - y\hat{j} - 4zk$$

at point $(1,2,1)$

$$\nabla \times B = 5(1)\hat{i} - 2\hat{j} - 4(1)k$$

$$\nabla \times B = 5i - 2j - 4k$$

(iv) grad div $A = \nabla \cdot (\nabla A) \cdot \nabla^2 A$

$$\text{grad div } A = \frac{d}{dx} \cdot \frac{dA}{dx} i + \frac{\partial}{\partial y} \cdot \frac{\partial A}{\partial y} + \frac{\partial}{\partial z} \frac{\partial A}{dz} k$$

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$\text{grad div } A = (2y + 1 + 2z)i + (2x)j + (1 + 2x)k$

$\text{grad div } A = (1 2(2) + 1 (2(1)))i + 2(1)j + (1 + 2(1))k$

$\text{grad div } A = 7i + 2j + 3k$

(4) Curl curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix} = i \begin{vmatrix} \frac{d}{dy} & \partial/\partial z \\ xy+yz & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+yz) \end{vmatrix}$$

$$\text{Curl } A = i\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right] - j\left[\frac{d}{dx}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right] + k\left[\frac{d}{dx}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$\text{Curl } A = yi - z^2j + (y - x^2)k$

$$\text{Curl curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial x \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$\text{Curl curl } A = i\left[\frac{d}{dy}[y - x^2] - \frac{d}{dz}(-z^2)\right] - j\left[\frac{d}{dx}(y - x^2) - \frac{d}{dz}(-y)\right] + k\left[\frac{d}{dx}(-z)^2 - \frac{d}{dy}(-y)\right]$$

$\text{Curl curl } A = i(1 + 2z) - j(-2x + 0) + k[0 + 1]$

$\text{Curl curl } A = (1 + 2z)i + 2xj + k$

$\text{Curl curl } A(1,2,1) = (1 + 2(1))j + 2(1)j + k$

$\text{Curl curl } A = 3i + 2j + k$