16/ENG 06/063

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MECHANICAL ENGINEERING

ENG 282

Question 1

i) Define mathematical modelling.

ii) Outline two methods of obtaining models for engineering systems

soln.

i) A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed "mathematical modelling"

It is also the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

ii) → using the balance Law.

→ forming a differential equation from an existing algebraic equation of the system.

Question 2

If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$,

determine i) $\frac{dr}{dt}$ ii) $\frac{d^2r}{dt^2}$ and

iii) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$.

soln

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$ //

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$= \sqrt{(2)^2 + (18\sin^2(0\times 3)) + (12e^{2\times 0})^2}$

$= \sqrt{4+0+144} = \sqrt{148}$

$= 12.166$ //

Question 3

If $A = x^2y\,i + (xy + yz)j + xz^2k$,
$B = yz\,i - 3xz\,j + 2xy\,k$, and
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$,
determine, at the point $(1, 2, 1)$

i) $\nabla\phi$,
ii) $\nabla \cdot A$,
iii) $\nabla \times B$,
iv) grad div A, and
vi) curl curl A.

soln

i) $\nabla\phi = \text{grad}\,\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\frac{\partial\phi}{\partial x} = 6xy + yz$

$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial\phi}{\partial z} = xy + 8y^2z$

$$\bar{\nabla}\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy + 8y^2z)k$$

$\bar{\nabla}\phi$ at point $(1, 2, 1)$

$$\bar{\nabla}\phi = [6(1)(2) + (2)(1)]i + [3(1)^2 + (1)(1) - 8(2)(1)^2]j + ((1)(2) + 8(2)^2(1)]k$$

$$\bar{\nabla}\phi = [12 + 2]i + [3 + 1 - 16]j + [2 - 32]k$$

$$\bar{\nabla}\phi = 14i - 12j - 30k$$

$$\Rightarrow (14, -12, -30)$$

ii) $\bar{\nabla} \cdot A = \operatorname{div} A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (a_x i + a_y j + a_z k)$

$$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$\bar{\nabla} \cdot A = 2xy + x + z + 2xz$$

at point $(1, 2, 1)$

$$\bar{\nabla} \cdot A = 2(1)(2) + (1) + (1) + 2(1)(1)$$

$$\bar{\nabla} \cdot A = 4 + 2 + 2 = 8$$

iii) $\bar{\nabla} \times B$

$$= \begin{vmatrix} i & \bar{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \left[\frac{\partial(2xy)}{\partial y} - \frac{\partial(-3xz)}{\partial z}\right]i - \left[\frac{\partial(2xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right]j + \left[\frac{\partial(-3xz)}{\partial x} - \frac{\partial(yz)}{\partial y}\right]$$

$$= [2x + 3x]i - [2y - y]j + [-3z - z]k$$

$$= 5xi - yj - 4zk$$

at point $(1, 2, 1)$

$$\bar{\nabla} \times B = 5(1)i - (2)j - 4(1)k$$

$$= 5i - 2j - 4k \Rightarrow (5, -2, -4)$$

iv) grad div A

$A = x^2 y i + (xy + yz) j + xz^2 k$

$\operatorname{div} A = 2xy + (x + z) + 2xz$

let $\operatorname{div} A = P$

$\therefore$ grad div A = grad P

$$\bar{\nabla} P = \frac{\partial P}{\partial x} i + \frac{\partial P}{\partial y} j + \frac{\partial P}{\partial z}$$

$$\bar{\nabla} P = (2y + 1 + 2z) i + (2x) j + (1 + 2x) k$$

at point (1, 2, 1)

$$\bar{\nabla} P = [2(2) + 1 + 2(1)] i + 2(1) j + (1 + 2(1)) k$$

$$\bar{\nabla} P = 7i + 2j + 3k \quad \Rightarrow (7, 2, 3)$$

v.) curl curl A

$A = x^2 y i + (xy + yz) j + xz^2 k$

$$\operatorname{curl} A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & [xy + yz] & xz^2 \end{vmatrix}$$

$$= \left[\frac{\partial (xz^2)}{\partial y} - \frac{\partial (xy + yz)}{\partial z}\right] i - \left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2 y)\right] j + \left[\frac{\partial [xy + yz]}{\partial x} - \frac{\partial}{\partial y}(x^2 y)\right] k$$

$$= [0 - y] i - [z^2 - 0] j + [y - x^2] k$$

at (1, 2, 1)

$$\Rightarrow -(2) i - [(1)^2 - 0] j + [2 - 1^2] k$$

$$= -2i - j + k$$

$\text{curl} A = -yi - z^2 j + (y - x^2)k$

let $\text{curl} A = P$

$\text{Curl curl } A = \text{curl } P$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y-x^2 \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right]i - \left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right]j$$

$$+ \left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]k.$$

$$= [1 - (-2z)]i - [2x - 0]j + [+\cancel{2}+1]k$$

at point $(1, 2, 1)$

$$\text{curl curl} A = \text{curl} P = [1 + (2(1))]i - 2(1)j + (1)k$$

$$= 3i + 2j + k \Rightarrow (3, 2, 1)$$