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16/ENG03/053
Civil Engineering

1) Mathematical modelling is the process of setting up a model solving it mathematically and interpreting the result in physical or in other terms.

ii a) using Balance law - Law of conservation of mass
b) Forming a differential equation from an existing algebraic equation of the system.

(2)

$r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t + 6e^{2t}k$

ii $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$= 2i + 18\sin 3t j + 12e^{2t}k$

iii $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 12k$

(3) if

$A = x^2yi + (xy + yz)j + xz^2k$.

$B = yzi - 3xzj + 2xyk$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$.

determine, at point $(1, 2, 1,)$

$\partial \nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$

$\nabla \phi = (6xy + yz) i + (3x^2 + xz - 8yz^2) j + (xy - 4y^2 2z$

at point $(1, 2, 1)$

$\nabla \phi = (6(1)(2) + (2)(1)) i + ((3(1)^2 + (1)(1) - 8(2)(1)^2) j$

$+ ((1)(2) - 4(2)^2 2(1)) k$

$\nabla \phi = 14 i - 12 j - 30 k$

ii) $\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z} \right) \cdot a_x i + a_y j + a_z k$

$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$= \frac{\partial}{\partial x} x^2 y) + \frac{\partial}{\partial y} (xy + yz) + \frac{\partial}{\partial z} (xz^2)$

$= 2xy + (x + z) + x2z$

$= 2(1)(2) + (1 + 1) + (1)2(1)$

$= 4 + 2 + 2$

at $(1, 2, 1)$

$\nabla \cdot A = 4 + 2 + 2$

$\nabla \cdot A = 8$

iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3x & 2xy \end{vmatrix}$

$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$

$i (\partial/\partial y (2xy) - \partial/\partial z (-3xz)] - j [\partial/\partial x (2xy) - \partial/\partial z (yz)]$

$+ k [\partial/\partial x (-3xz) - \partial/\partial y (yz)]$

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$\nabla \times B = i\,[2z+3x] - j\,[2y-y] + k\,[-3z-z]$

at Point $(1,2,1)$

$\nabla \times B = i\,(2(1)+3(1)) - j\,[2(2)-(2)] + k\,[-3(1)-(1)]$

$= 5i - 2j - 4k$

(iv) grad div A

$\text{div } A = \left(\frac{\partial i}{\partial x} + \frac{\partial i}{\partial y} + \frac{\partial x}{\partial z}\right) \cdot a_x i + a_y j + a_z k$

$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$= \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z} xz^2$

$= 2xy + (x+z) + x2z = A$

$\text{grad Div } A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (2xy + (x+z) + x2z)$

$= (2y + z + 2z)\,i + j(2x) + (1+2)k$

at $(1,2,1)$

$\text{grad Div } A = (2(2) + (1) + 2(1))\,i + (2(1))\,j + (1+2)k$

$\nabla \nabla \cdot A = 7i + 2j + 3k$

(v) Curl curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy+yz) \end{vmatrix}$$

$i\,[0 - y] - j\,[z^2 - 0] + k\,(y - x^2)$

$= -y\,i - z^2 j + (y - x^2)\,k$

$+ k\left[\partial/\partial x\,(-z^2) - \partial/\partial y\,(-y)\right]$

$\text{curl curl } A = i\,[1+2z] - j\,[-2x+0] + k\,(0+1)$

$\text{curl curl } A = (1+2z)\,i + 2xj + k$

at Point $(1,2,1)$

$\text{Curl Curl } A = (1 + 2(1))\,i + 2(1)\,j + k$

$= 3i + 2j + k$

(1) ii $\left|\frac{d^2 r}{dt^2}\right|$ at $t = 0$

$\frac{d^2 r}{dt^2} = 2i + 18\sin 3(0)\,j + 12e^{2(0)}$

$t=0 \quad 2i + 12k$

$\left|\frac{d^2 r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$

$= 12.165$

≈ 12.17