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matric no: 16/eng04/009
course: eng 282

NO 1

1(a) mathematical modelling can be defined as the process of setting up a model of an engineering problem, solving it mathematically and interpreting the result in physical or other terms.

This model is the formulation of the problem as a mathematical expression in terms of variables, functions and equations.

(ii) Two methods of obtaining models for engineering system

① Abstraction and scaling

② ~~convert~~ conservation and Balance principles.

NO 2

$$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{3t} k$$

(i) $$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{3t} k$$

(ii) $$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{3t} k$$

(iii) at $t=0$ $$\frac{d^2r}{dt^2} = 2i + 18\sin(3\times 0)\, j + 12e^{2(0)} k$$

$$= 2i + 12k$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2+(12)^2} = \sqrt{4+144} = \sqrt{148}$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$$

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NO 3

$A = x^2 y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

at the point (1, 2, 1)

(i) $\nabla \phi = \text{grad } \phi = \left(\frac{d i}{dx} + \frac{d j}{dy} + \frac{d k}{dz} \right) \cdot (3x^2 y + xyz - 4y^2 z^2 - 3)$

$= \frac{d}{dx}(3x^2 y + xyz - 4y^2 z^2 - 3) i + \frac{d}{dy}(3x^2 y + xyz - 4y^2 z^2 - 3) j$

$+ \frac{d}{dz}(3x^2 y + xyz - 4y^2 z^2 - 3) k$

$= i(6xy + yz) + j(3x^2 + xz - 8yz^2) + k(xy - 8y^2 z)$

at the point (1, 2, 1)

$\nabla \phi = i[6(1)(2) + (2)(1)] + j[3(1)^2 + (1)(1) - 8(2)(1)^2] +$
$k[(1)(2) - 8(2)^2(1)]$

$= i(12 + 2) + j(3 + 1 - 16) + k(2 - 32)$

$= 14i - 12j - 30k$

(ii)

(ii) $\text{div } A = \nabla \cdot A = \left(\frac{d i}{dx} + \frac{d j}{dy} + \frac{d k}{dz} \right) \cdot (x^2 y i + (xy + yz) j + xz^2 k)$

$\nabla \cdot A = \frac{d}{dx}(x^2 y) + \frac{d}{dy}(xy + yz) + \frac{d}{dz}(xz^2)$

$= 2xy + (x + z) + 2xz$

at (1, 2, 1)

$= 2(1)(2) + (1 + 1) + 2(1)(1) = 4 + 2 + 2$

$\nabla \cdot A = 8$

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(III)

III) curl B = $\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ y^2 & -3xz & 2xy \end{vmatrix}$

$\nabla \times B = i\left[\frac{d}{dy}(2xy) + \frac{d}{dz}(3xz)\right] - j\left[\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right]$

$+ k\left[\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right]$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

at (1, 2, 1)

$\nabla \times B = 5(1)i - (2)j - 4(1)k$

$= 5i - 2j - 4k$

(IV)

IV) grad div A = $\nabla(\nabla \cdot A)$

$\nabla \cdot A = \left(\frac{di}{dx} + \frac{dj}{dy} + \frac{dk}{dz}\right) \cdot x^2yi + (xy + yz)j + xz^2k$

$= \frac{d}{dx}(x^2y) + \frac{d}{dy}(xy + yz) + \frac{d}{dz}(xz^2)$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$\nabla(\nabla \cdot A) = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right) \cdot 2xy + (x + z) + 2xz$

$= \frac{d}{dx}(2xy + (x+z) + (2xz))i + \frac{d}{dy}[2xy + (x+z) + 2xz]j$

$+ \frac{d}{dz}(2xy + (x+z) + 2xz)k$

$= i(2y + 1 + 2z) + (2x)j + (1 + 2x)k$

at (1, 2, 1)

$\nabla(\nabla\cdot A) = [(4)(2) + 1 + 2(1)]i + 2(1)j + [1+2(1)]k$

$= (4+1+2)i + 2j + (1+2)k$

$= 7i + 2j + 3k$

(V)

v) curl curl $A = \nabla\times(\nabla\times A)$

$$\nabla\times A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right] - j\left[\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right] + k\left[\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right]$$

$\nabla\times A = i(-y) - j(z^2) + k(y - x^2)$

$$\nabla\times(\nabla\times A) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left[\frac{d}{dy}(y-x^2) + \frac{d}{dz}(z^2)\right] + j\left[\frac{d}{dx}(y-x^2) + \frac{d}{dz}(y)\right] + k\left[\frac{d}{dx}(-z^2) + \frac{d}{dy}(y)\right]$$

$\nabla\times(\nabla\times A) = i(1-2z) + j(-2x) + k(1)$

$=$ at $(1,2,1)$

$= i(1+2(1)) + j(-2(1)) + k(1)$

$= i(1+2) + j(-2) + k$

$\nabla\times(\nabla\times A) =$ ~~3i - 2j + k~~ $3i + 2j + k$

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