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16/ENG02/032
COMPUTER ENGR
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ANSWERS.

QUESTION 1

(i) Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application

(a) (i) In chemical engineering → Chemical equilibrium

(ii) In computer engineering → technology optimization.

QUESTION 2.

$r = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$

$\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

$\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

$\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$

$= 2\hat{i} + 18\sin(3\times 0)\hat{j} + 12e^{2(0)}\hat{k}$

$= 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2} = \sqrt{4+144}$

$= \sqrt{148}$

$= 2\sqrt{37}$

$= 12.1655$

$= 12.17$

QUESTION 3.

$A = x^2y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$

$B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(c) $\nabla\phi$ at point $(1,2,1)$

$\nabla\phi = \frac{\delta\phi}{\delta x}\hat{i} + \frac{\delta\phi}{\delta y}\hat{j} + \frac{\delta\phi}{\partial z}\hat{k}$

$\frac{\delta\phi}{\delta x} = 6xy + yz$

$\frac{\delta\phi}{\delta y} = 3x^2 + xz - 8yz^2$

$\frac{\delta\phi}{\delta z} = xy - 8y^2z$

$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2z)\hat{k}$

at point $(1,2,1)$

$\nabla\phi = (12+2)\hat{i} + (3+1-8(2)(1))\hat{j} + (2-32)\hat{k}$

$= 14\hat{i} + (4-16)\hat{j} + (-30)\hat{k}$

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$

(ii) $\nabla \cdot A = \frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k} \left(x^2y\hat{i} + (xy+yz^2)\hat{j} + xz^2\hat{k}\right)$

$\nabla \cdot A = \frac{\delta}{\delta x}(x^2y) + \frac{\delta}{\delta y}(xy+yz) + \frac{d}{\delta z}(xz^2)$

$\nabla \cdot A = 2xy + (x+z) + 0 + (2xz)$
$= 2(1)(2) + (1+1) + 2\times1\times1 = 6+2$
$= 4+2 = 6+2 = 8$

$\nabla \cdot A = 8$

(iii) $\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ yz & -3xz & 2xy \end{vmatrix}$

$\hat{i}\left(\frac{\delta}{\delta y}(2xy) + \frac{\delta(3xz)}{\delta z}\right) - \hat{j}\left(\frac{\delta(2xy)}{\delta x} - \frac{\delta(yz)}{\delta z}\right) + \hat{k}$

$\left(\frac{\delta}{\delta z}(-3xz) - \frac{\delta}{\delta y}(yz)\right)$

$\hat{i}(2x+3x) - \hat{j}(2y-y) + \hat{k}(-3z-z)$
$\hat{i}(5x) - \hat{j}(y) + \hat{k}(-4z)$

$5x\hat{i} - y\hat{j} - 4z\hat{k}$ at point $(1,2,1)$
$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

(iv) grad div A $\nabla \cdot \vec{A}$

div $A = 2xy + (x+z) + 2zx$
$\nabla \cdot (\nabla A) = \frac{\delta(\nabla A)}{\delta x}\hat{i} + \frac{\delta(\nabla A)}{\delta y}\hat{j} + \frac{\delta(\nabla \cdot A)}{\delta z}\hat{k}$

$\therefore \nabla(\nabla\cdot A) = \hat{i}\,(2y+1+2z) + 2x\hat{j} + (1+2x)\hat{k}$

$= \hat{i}\,(4+1+2) + 2\hat{j} + 3\hat{k}$

$= 7\hat{i} + 2\hat{j} + 3\hat{k}.$

(v) $\nabla\times(\nabla\times\vec{A})$ $\quad \vec{A} = x^2y\hat{i} + (xy+yz)\hat{j} + xz^2\hat{k}$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ x^2y & (xy+yz) & (xz^2) \end{vmatrix}$$

$\hat{i}\,(0-y) - \hat{j}\,(z^2-0) + \hat{k}\,(x^2+(-y))$

$-y\hat{i} - \hat{j}\cdot(z^2) + \hat{k}\cdot(x^2-y)$

$\nabla\times\vec{A} = -y\hat{i} - \hat{j}\,(z^2) + \hat{k}\,(x^2-y)$

$$\therefore \nabla\times(\nabla\times\vec{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ -y & -z^2 & x^2-y \end{vmatrix}$$

$\hat{i}\,(1+2z) - \hat{j}\,(0-2x) + \hat{k}\,(1-0)$

$\nabla\times(\nabla\times\vec{A}) = (1+2z)\hat{i} + 2x\hat{j} + \hat{k}$

$= (1+2(1))\hat{i} + 2\hat{j} + \hat{k}$

$\nabla\times(\nabla\times\vec{A}) = 3\hat{i} + 2\hat{j} + \hat{k}$ //

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