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1, mathematical models defined as a process of developing a mathematical model.
methods of developing model in engineering system
1, Radioactivity
2, mixing problems

Question 2

$r = (t^2 + 3t)i - 3\sin 3t\, j + 3e^{3t}k$

a, $\frac{dr}{dt} = (2t + 3)i - (6\cos 3t)j + (6e^{2t})k$

b, $\frac{d^2r}{dt^2} = (2)i + (18\sin 3t)j + (12e^{2t})k$

$\frac{d^2r}{dt^2}$ at $t = 0$ $= 2i + 18\sin 0\, j + 12e^{0}k$

$\frac{d^2r}{dt^2} = 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$

$= \sqrt{4 + 144}$

$= \sqrt{148}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.2$ units

Question 3

$A = x^2 y i + (x^2y + yz) j + x z^2 k$

$B = yz i - 3xz j + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 \; 5$

$\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right]\phi$

$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$

$\nabla\phi = i\,[yz + 6xy] + j\,[3x^2 + xz - 8yz^2] + k\,[xy - 8y ...$

at point $(1,2,1)$

$\nabla\phi = i\,[2\times1 + 12] + j\,[3\times1^2 + 1\times1 - 8\times2\times1^2] + k\,[1\times2 - 8\times ...$

$\nabla\phi = i(14) + j(-12) + k(-30)$

$\nabla\phi = 2i - 12j + 30k$

$\nabla \cdot A$

$\nabla = i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

$A = a_x i + a_y j + a_z k$

$\nabla \cdot A = \left[i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right] \cdot \left[a_x i + a_y j + a_z k \right]$

$\nabla \cdot A = \frac{\partial}{\partial x} \cdot a_x + \frac{\partial}{\partial y} \cdot a_y + \frac{\partial}{\partial z} \cdot a_z$

$2xy + (x + z) + 2xz$

$\nabla \cdot A$ at $(1,2,1)$

$\nabla \cdot A = 2\times1\times2 + (1+1) + 2\times1\times1$

$\nabla \cdot A = 4 + 2 + 2$

$\nabla \cdot A = 8$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\left|\begin{matrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -3xz & 2xy\end{matrix}\right| - j\left|\begin{matrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ yz & 2xy\end{matrix}\right| + k\left|\begin{matrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ y^2 & -3xz\end{matrix}\right|$$

$$i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right] +$$

$$k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(y^2\right.$$

$= (2x + 3x)i - j(2y - y) + k(-3z - z]$

$\nabla \times B$ at $(1, 2, 1)$

$\nabla \times B = (2\times1 + 3\times1)i - j(2\times2 - 2) + k(-3\times1 - 1)$

$\nabla \times B = (5)i - j(2) + k(-4)$

$\nabla \times B = 5i - 2j - 4k.$

Grad of div A

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$\nabla A = \left[\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right] \nabla \cdot A$

$\nabla \cdot A = [(2y + 1 + 2z)i + j(2x) + (1 + 2x)k]$

$\nabla(\nabla \cdot A)$ at $(1, 2, 1)$

$\nabla(\nabla \cdot A) = (2\times2 + 1 + 2\times1)i + (2\times1)j + (1\times2\times1)k$

$= (7)i + (2)j + 3k$

$7i + 2j + 3k$

Curl curl A

$$\text{Curl } A = \nabla \times A = \left|\begin{matrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2y & (xy+yz) & (xz^2)\end{matrix}\right|$$

$$\left|\begin{matrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (xy+yz) & (xz^2)\end{matrix}\right| - j\left|\begin{matrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ x^2y & (xz^2)\end{matrix}\right| + k\left|\begin{matrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ x^2y & (xy+yz)\end{matrix}\right|$$

$$\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right) - j\left\{\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right\}$$

$$+ k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$i[0-y] - j\{z^2 - 0\} + k\,[y - x^2]$$

$$= -yi - z^2 j + (y-x^2)k$$

$$\nabla\times(\nabla\times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$i\left|\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right| - j\left|\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right|$$

$$+ k\left|\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right|$$

$$i(1+2z) - j(-2x+0) + k(0+1)$$

$\nabla\times(\nabla\times A)$ at $(1,2,1)$

$$\nabla\times(\nabla\times A) = i(1+2\times1) - j(2\times1) + k(1)$$

$$\nabla\times(\nabla\times A) = 3i + 2j + k$$