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1. A mathematical model is a description of a System using mathematical Concepts and language. Therefore modelling is the process of Setting up a model, Solving it mathematically and interpreting the result in physical and Other terms.

b

i Exponential growth / decay (use of ODE)

ii Mixing problems.

2. $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} K$

i $\frac{dr}{dt} = (2t + t)\hat{i} - 6\cos 3t \hat{j} + 6e^{2t} \hat{k}$

ii $\frac{d^2r}{dt^2} = 2\hat{i} + 18 \sin 3t \hat{j} + 12e^{2t} \hat{k}$

iii $\frac{d^2r}{dt^2}$ at $t=0$ $= 2\hat{i} + 12\hat{k}$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37} = 12.17$$

3. $A = x^2 y \hat{i} + (xy + yz) j + xz^2 K$

$B = YZi - 3xzj + 2xyK$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3.$

i $\nabla \phi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$

$\frac{\partial \phi}{\partial x} = 6xy + yz$ $\quad \frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial \phi}{\partial z} = xy - 2y^2 z$

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$A\ \overset{x}{(1,}\ \overset{y}{2,}\ \overset{z}{1)}$

$$\frac{\partial\phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\partial\phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12.$$

$$\frac{\partial\phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$$

ii $\nabla\cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\nabla\cdot A = 2xy + (x+z) + 2xz$

$A + (1, 2, 1)$

$\nabla\cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii $\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z).$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

$A + (1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv Gradient A

$\cdot$ grad$(2xy + (x+z) + 2xz)$

let div $A = C = \nabla A$

$\nabla(\nabla A) = \nabla C = \hat{i}\frac{\partial c}{\partial x} + \hat{j}\frac{\partial c}{\partial y} + \hat{k}\frac{\partial c}{\partial z}$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1 + 2x)$

$A + (1, 2, 1)$

$$\nabla C = i(2(2)+1+2(1)) + \hat{j}(2(1)) + \hat{k}(1+(2)(1)]$$
$$= i(4+1+2) + \hat{j}(2) + \hat{k}(1+2)$$
$$= 7\hat{i} + 2\hat{j} + 3\hat{k}$$

V. Curl Curl A

Curl $A = \nabla A$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0-y) - \hat{j}(z^2-0) + \hat{k}(y-x^2)$$
$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y-x^2)$$

At $(1,2,1)$

Curl $A = -2\hat{i} - \hat{j} + \hat{k}$

Curl Curl $A = \nabla\times(\nabla\times A)$

$$\nabla\times(\nabla\times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}(1+2z) - \hat{j}(-2x-0) + \hat{k}(0+1)$$
$$= \hat{i}(1+2z) + 2x\hat{j} + \hat{k}$$

At Point $(1,2,1)$

$$\nabla\times(\nabla\times A) = \hat{i}(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$$
$$= 3\hat{i} + 2\hat{j} + \hat{k}$$