

1 a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting results in physical and other terms.

b) Exponential growth/decay (use of DDE)
 (ii) Mixing problems.

2) $r = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$
 i) $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$
 ii) $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii) $\left. \frac{d^2r}{dt^2} \right|_{t=0} = 2\hat{i} + 12\hat{k}$
 $\left| \frac{d^2r}{dt^2} \right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144}$
 $= \sqrt{148}$
 $= 2\sqrt{37}$
 $= 12.17$

3) $A = x^2y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$
 $B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$
 $\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$$i) \quad \psi = \frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$$

$$\frac{\partial \phi}{\partial x} = 6xy + yz$$

$$\frac{\partial \phi}{\partial z} = xy - 8y^2z$$

$$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$$

$$\text{At } (1, 2, 1)$$

$$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$$

$$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2) + (1) = 2 - 16 + 1 = -13$$

$$\nabla \phi = 14\hat{i} - 12\hat{j} - 13\hat{k}$$

$$ii) \quad \nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$A = x\hat{i} + 2y\hat{j} + z\hat{k}$$

$$\nabla \cdot A = 2xy + (x+2) + 2xz$$

$$\text{At } (1, 1, 1)$$

$$\nabla \cdot A = 2(1)(1) + (1+1) + 2(1)(1) = 2 + 2 + 2 = 6$$

$$iii) \quad \nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\begin{aligned}
 &= \hat{i}(2x+3x) - \hat{j}(2y-y) + \hat{k}(-3z-z) \\
 &= 5x\hat{i} - 3y\hat{j} - 4z\hat{k} \\
 \text{At } (1, 2, 1) \\
 \nabla \times B &= 5\hat{i} - 2\hat{j} - 4\hat{k}
 \end{aligned}$$

iv) grad u

$$\text{grad}(2xy + (x+z) + 2xz)$$

$$\text{let } \text{div } A = C = \nabla A$$

$$\begin{aligned}
 \nabla(\nabla A) &= \nabla C = \hat{i} \frac{dC}{dx} + \hat{j} \frac{dC}{dy} + \hat{k} \frac{dC}{dz} \\
 &= \hat{i}(2y+1+2z) + \hat{j}(2x) + \hat{k}(1+2x)
 \end{aligned}$$

$$\text{At } (1, 2, 1)$$

$$\begin{aligned}
 \nabla C &= \hat{i}(2(2)+1+2(1)) + \hat{j}(2(1)) + \hat{k}(1+2(1)) \\
 &= \hat{i}(4+1+2) + \hat{j}(2) + \hat{k}(1+2) \\
 &= 7\hat{i} + 2\hat{j} + 3\hat{k}
 \end{aligned}$$

v)

$$\begin{aligned}
 A &= \nabla \times A \\
 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix} \\
 &= \hat{i}(0-y) - \hat{j}(2^2-0) + \hat{k}(y-x^2) \\
 &= -y\hat{i} - 2^2\hat{j} + \hat{k}(y-x^2) \\
 \text{At } (1, 2, 1)
 \end{aligned}$$

$$\text{Curl } A = -2\hat{i} - \hat{j} + \hat{k}$$

$$A = \nabla \times (\nabla \times A)$$

$$\begin{aligned}
 \nabla \times (\nabla \times A) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -2^2 & (y-x^2) \end{vmatrix} \\
 &= \hat{i}(1+2z) - \hat{j}(-2x-0) + \hat{k}(0+1) \\
 &= \hat{i}(1+2z) + 2x\hat{j} + \hat{k}
 \end{aligned}$$

At point (1, 2, 1)

$$\nabla \times (\nabla \times A) = \hat{i}(1 + 2(1)) + 2(1)^2 \hat{j} + \hat{k}$$
$$= 3\hat{i} + 2\hat{j} + \hat{k}$$