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Mechatronics Engineering
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ENG 282
200 Level

I) Mathematical modelling is a description of a system using mathematical concepts and language

II) - Modelling
- Simulation

2) $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

I) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, j + 6e^{2t} k$

II) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k$

III) $\frac{d^2r}{dt^2}$ at $t = 0$

$= (2i + 18\sin 3(0)j + 12e^{2(0)}k)$

$= 2i + 12k$

$= \sqrt{2^2 + 12^2}$

$= \sqrt{148} = 2\sqrt{37}$

3) I) $\nabla\phi$

$= \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) 3x^2y + xyz - 4y^2z^2 - 3$

$= \frac{\partial}{\partial x}i\ 3x^2y + \frac{\partial}{\partial x}i\ xyz - \frac{\partial}{\partial x}i\ 4y^2z^2 - \frac{\partial}{\partial x}i\ 3 + \frac{\partial}{\partial y}j\ 3x^2y$

$+ \frac{\partial}{\partial y}j\ xyz - \frac{\partial}{\partial y}j\ 4y^2z^2 - \frac{\partial}{\partial y}j\ 3 + \frac{\partial}{\partial z}k\ 3x^2y$

$+ \frac{\partial}{\partial z}k\ xyz - \frac{\partial}{\partial z}k\ 4y^2z^2 - \frac{\partial}{\partial z}k\ 3$

$= 6xyi + yzi - 0 - 0 + 3x^2j + xzj - 8yz^2j$

$- 0 + 0 + xyk - 8y^2zk - 0k$

$= 6xyi + yzi + 3x^2j + xzj - 8yz^2j + xyk$

$- 8y^2zk$

at $(1, 2, 1)$

$= 6(1\times2)i + (2\times1)i + 3(1)^2j + (1\times1)j - 8(2\times1^2)j$

$+ (1\times2)k - 8(2)^2(1)k$

$= 12i + 2i + 3j + 1j - 16j + 3k - 32k$

$\nabla\phi = 14i - 12j - 27k$

II) $\nabla \cdot A$

$= \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) \cdot x^2yi + (xy + yz)j + xz^2k$

$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}xz^2$

$= 2xy + x + z + 2xz$

$= 2(1)(2) + 1 + 1 + 2(1)(1)$

$= 4 + 1 + 1 + 2$

$= 8$

III) $\nabla \times B$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \left(\frac{d\,2xy}{dy} - \frac{d(-3xz)}{dz}\right)i - \left(\frac{d\,2xy}{dx} - \frac{dyz}{dz}\right)j + \left(\frac{d(-3xz)}{dx} - \frac{dyz}{dy}\right)k$$

$= (2x + 3x)i - (2y - y)j + (-3z - z)k$

$= 5xi - yj - 4zk$

$= 5(1)i - 2j - 4(1)k$

$= 5i - 2j - 4k$

IV) grad div A.

$|A| = (1)^2(2)i + (1)(2) + (2)(1)j + (1)(1)^2 k$

$|A| = 2i + 4j + k$

$|A| = \sqrt{2^2 + 4^2 + 1^2}$

$|A| = \sqrt{21}$

V)

IV) Grad div A

$$\text{div } A = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) \cdot x^2y\,i + (xy+yz)\,j + xz^2\,k$$

$$= 2xy + x + z + 2xz$$

$$\text{grad div } A = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) \cdot 2xy + x + z + 2xz$$

$$= \frac{\partial(2xy + x + z + 2xz)}{\partial x}i + \frac{\partial(2xy + x + z + 2xz)}{\partial y}j$$

$$\frac{\partial(2xy + x + z + 2xz)}{\partial z}k$$

$$= 2yi + i + 2zi + 2xj + k + 2xk$$

$$= 4x + 2y + 2z + 2$$

$$= (2y + 1 + 2z)i + 2xj + (1 + 2x)k$$

$$= (2(2) + 1 + 2(1))i + 2(1)j + (1 + 2(1))k$$

$$= (4 + 1 + 2)i + 2j + 3k$$

$$= 7i + 2j + 3k$$

V) curl curl A

$\nabla \times \nabla \times A$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \left(\frac{\partial xz^2}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right)i - j\left(\frac{\partial xz^2}{\partial x} - \frac{\partial x^2y}{\partial z}\right)$$

$$+\left(\frac{\partial(xy+yz)}{\partial x}-\frac{\partial x^2y}{\partial y}\right)k$$

$$=(0-0-y)i-j(z^2-0)j+(y-x^2)k$$

$$=yi-z^2j+(y-x^2)k$$

$$\nabla\times A=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z^2 & (y-x^2) \end{vmatrix}$$

$$=\left(\frac{\partial(y-x^2)}{\partial y}-\frac{\partial z^2}{\partial z}\right)i-\left(\frac{\partial(y-x^2)}{\partial x}-\frac{\partial y}{\partial z}\right)j$$

$$+\left(\frac{\partial z^2}{\partial x}-\frac{\partial y}{\partial y}\right)k$$

$$=(1-2z)i-(-2x)j+(-1)k$$

$$=(1-2(1))i-(-2(1))j-k$$

$$=-i+2j-k$$