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16/ENG04/1034
Electrical Electronics Engineering
Course Code: ENG 282

Assignment / Answer

(a) A mathematical model is a description of a system using mathematical ~~Concepts~~ Concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms

b i) Exponential growth / decay (use of ODE)
ii) Mixing problems.

2) $r = (t^2 + 3t)i - 2\sin 3tj + 3e^{2t}k$

i) $\frac{\partial r}{\partial t} = (2t+3)i - 6\cos 3tj + 6e^{2t}k$

ii) $\frac{\partial^2 r}{\partial t^2} = 2i + 18\sin 3tj + 12e^{2t}k$

iii) $\left.\frac{\partial^2 r}{\partial t^2}\right|_{t=0} = 2i + 12k$

$\left|\frac{\partial^2 r}{\partial t^2}\right| = \sqrt{2^2+12^2} = \sqrt{4+144} = \sqrt{148}$
$= 2\sqrt{37} = 12.17$

) $A = x^2yi + (xy+y^2)j + xz^2k$
$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

) $\bar{V}\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$

$\frac{\partial \phi}{\partial x} = 6xy + yz$

$\frac{\partial \phi}{\partial z} = xy - 8y^2 z$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At (1, 2, 1)

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16$
$= -12$

~~$\nabla\phi = 14i - 12j$~~

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla\phi = 14i - 12j - 30k$

ii) $\bar{V} \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = axi + a_y j + a_z k$

$\bar{V} \cdot A = 2xy + (x + z) + 2xz$

At (1, 1, 1)

$\bar{V} \cdot A = 2(1)(2) + (1 + 1) + 2(1)(1)$
$= 4 + 2 + 2 = 8$

iii) $\bar{V} \times B$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x+3x) - j(2y-y) + k(-3z-z)$

$= 5xi - yj - 4zk$

At (1, 2, 1)

$\bar{V} \cdot B = 5i - 2j - 4k$

iv) grad div A

grad $(2xy + (x+2) + 2xz)$

let $div A = C = \nabla \cdot A$

$\nabla(\nabla \cdot A) = \nabla C = i\frac{\partial c}{\partial x} + j\frac{\partial c}{\partial y} + k\frac{\partial c}{\partial z}$

$= i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$

At (1, 2, 1)

$\nabla C = i(2(2) + 1 + 2(1)) + j(2(1)) + k(1+(2)(1))$

$= i(4+1+2) + j(2) + k(1+2)$

$= 7i + 2j + 3k$

v) Curl Curl A

Curl A $= \nabla \times A$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (x-y+z) & xz^2 \end{vmatrix}$$

$= i(0-y) - j(z^2-0) + k(y-x^2)$

$= -yi - z^2j + k(y-x^2)$

At (1, 2, 1)

Curl A $= -2i - j + k$

Curl Curl A $= \nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$= i(1+2z) - j(-2x-0) + k(0+1)$

$= i(1+2z) + 2x^2j + k$

At point (1, 2, 1)

$\nabla \times (U \times A) = i(1 + 2(1)) + 2(1)^2 j + k$

$= 3i + 2j + k$