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Course Engineering Maths II

ENG 282 Assignment 3.

- Mathematical models is defined as a process of developing a mathematical model.
Methods of developing model in engineering system
1) Radioactivity 2) Mixing problems.

Question 2.

1) $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t}k$

a.) $\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$

b.) $\frac{d^2r}{dt^2} = (2)i + (18\sin 3t)j + (12e^{2t})k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 18\sin 0 j + 12e^0 k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{148}$

$= 12.2$ units//

Question 3

$A = x^2y\,i + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz + 4y^2z^2 - 3$

$$\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]\phi$$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$\nabla\phi = i(yz + 6xy + j(3x^2 + xz - 8yz^2] + k(xy - 8y^2z)$$

at point $(1,2,1)$

$$\nabla\phi = 1[2\times1 + 12] + y(3\times1^2 + 1\times1 - 8\times2\times1^2) + k(1\times2 - 8\times2^2\times1]$$

$$\nabla\phi = 1(14) + j(-12) + k(-30)$$

$$\nabla\phi = 2i + 12j + 30k$$

∇A

$$\nabla = i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

$A = axi + ayj + azk$

$$\nabla\cdot A = \left[i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right]\cdot\left[axi + ayj + azk\right]$$

$$\nabla\cdot A = \frac{\partial}{\partial x}\cdot ax + \frac{\partial}{\partial y}\cdot ay + \frac{\partial}{\partial z}\cdot az$$

$$= 2xy + (x+z) + 2xz$$

$\nabla\cdot A$ at $(1,2,1)$

$$\nabla\cdot A = 2\times1\times2 + (1+1)\times2\times1\times1$$

$$\nabla\cdot A = 4 + 2 + 2$$

$$\nabla\cdot A = 8$$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3xz & 2xy \end{vmatrix} y\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ yz & -3xz \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - y\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(-yz)\right. +$$

$$k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(y^2\right.$$

$$= (2x + 3x)i - y(2y - -y) + k(-3z - z)$$

$\nabla \times B$ at $(1, 2, 1)$

$$\nabla \times B = (2\times1 + 3\times1)i\ y(2\times2 - 2) + k(-3\times1 - 1)$$

$$\nabla \times B = (5)i - j(2) + k(-4)$$

$$\nabla \times B = 5i - 2j - 4k$$

Grad of div A

$$\nabla \cdot A = 2xy + (x+z) + 2xz$$

$$\nabla A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\nabla \cdot A$$

$$= \left[(2y + 1 + 2z)i + j(2x) + (1 + 2x)k\right]$$

$\nabla(F\nabla \cdot A)$ at $(1, 2, 1)$

$$\nabla(\nabla \cdot A) = (2\times2 + 1 + 2\times1)i + (2\times1)j + (1\times2\times1)k$$

$$= (7)i + (2)j + 3k$$

$$7i + 2j + 3k$$

Calculate A

$$\text{Curl } A = \nabla \times A \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy+yz) & (xz^2) \end{vmatrix}$$

$$i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (xy+yz) & (xz^2) \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & (xz^2) \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & (xy+y^2 \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+y^2)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right)$$

$$+k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$i[0 - y] - j[z^2 - 0] + k[y - x^2]$$

$\nabla \times A = -y i - z^2 j + (y - x^2) k$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$$i \left[\frac{\partial}{\partial y}(y - x^2) - \frac{\partial}{\partial z}(-z^2) \right] - j \left(\frac{\partial}{\partial x}(y - x^2) - \frac{\partial}{\partial z}(-y) \right)$$

$$+ k \left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y) \right]$$

$i(1 + 2z) - j(-2x + 0) + k(0 + 1)$

$\nabla \times (\nabla \times A)$ at $(1, 2, 1)$

$\nabla \times (\nabla \times A) = i(1 \times 2 \times 1) - j(-2 \times 1] + k(1)$

$\nabla \times (\nabla \times A) = 3i + 2j + k$ //